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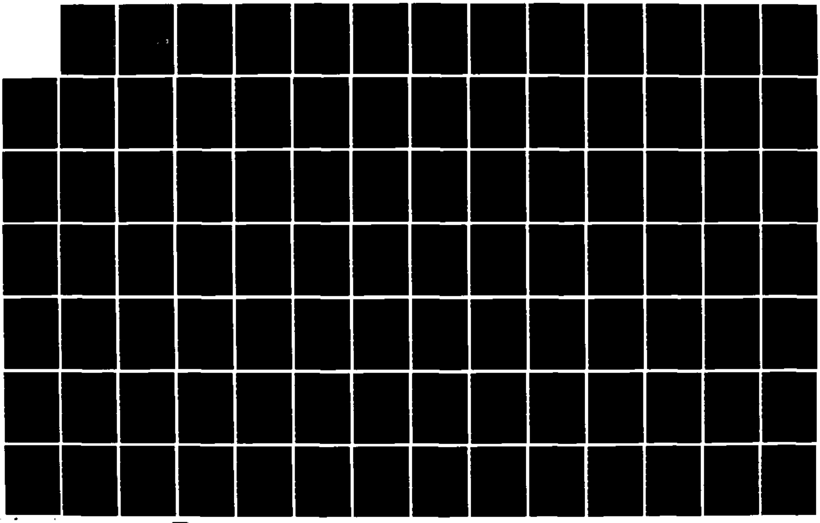

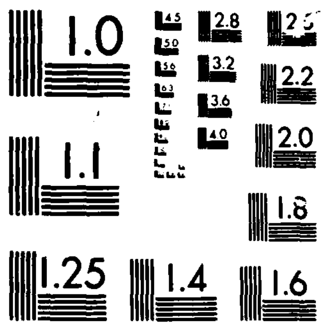

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SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE — READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER: AFIT/CI/NR 86-47T
2. GOVT ACCESSION NO.
3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle): Control of Large Flexible Systems by Spatial Modal Input-Distribution Control
5. TYPE OF REPORT & PERIOD COVERED: THESIS/~~DISSERTATION~~
6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s): Craig Vincent Bendorf
8. CONTRACT OR GRANT NUMBER(s)
9. PERFORMING ORGANIZATION NAME AND ADDRESS: AFIT STUDENT AT: The Ohio State University
10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS: AFIT/NR WPAFB OH 45433-6583
12. REPORT DATE: 1985
13. NUMBER OF PAGES: 102
14. MONITORING AGENCY NAME & ADDRESS (If different from Controlling Office)
15. SECURITY CLASS. (of this report): UNCLASS
15a. DECLASSIFICATION/DOWNGRADING SCHEDULE
16. DISTRIBUTION STATEMENT (of this Report): APPROVED FOR PUBLIC RELEASE; DISTRIBUTION UNLIMITED

DTIC ELECTE APR 23 1986 S D A

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)
18. SUPPLEMENTARY NOTES: APPROVED FOR PUBLIC RELEASE: IAW AFR 190-1

LYNN E. WOLAVER 16 April 86
Dean for Research and Professional Development
AFIT/NR, WPAFB OH 45433-6583

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)
20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

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DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE

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THESIS ABSTRACT

AUTHOR: Craig Vincent Bendorf

RANK: Captain BRANCH: USAF

DATE: 1985 PAGES: 102

DEGREE: Master of Science in Aeronautical and Astronautical Engineering

INSTITUTION: The Ohio State University

TITLE: Control of Large Flexible Systems by Spatial Modal Input-Distribution Control

A common problem in controlling a large flexible structure with a reduced order model is the excitation of the residual (non-modelled) modes by the control inputs resulting in control spillover. In this thesis a new approach to eliminate control spillover is examined. This approach, based on the independent modal-space control method, uses a finite number of spatially distributed input points to eliminate control spillover. Control of an undamped beam and a finite degree of freedom truss is accomplished through computer simulation. The results of the seven example problems shows that this new approach does effectively eliminate control spillover.

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CONTROL OF LARGE FLEXIBLE SYSTEMS BY SPATIAL

MODAL INPUT-DISTRIBUTION CONTROL

A Thesis

Presented in Partial Fulfillment of the Requirements for

the degree Master of Science in the Graduate

School of the Ohio State University

by

Craig Vincent Bendorf, B.S.

* * * * *

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1985

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ACKNOWLEDGEMENTS

I would like to express my sincere appreciation to Dr. Hayrani Öz for his help and guidance in preparing this thesis. Thanks go to Dr. Nesrin Sarigul for her suggestions and comments.

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INTRODUCTION

With the increased capabilities of the space shuttle man will now be able to build large structures in space. These structures will have applications in many areas that will demand precise pointing and control such as solar power stations, radio astronomy, and communications antennas. In order to satisfy mission performance requirements, these large structures will require control systems that will actively suppress the vibrations of the structure [1].

As a space structure increases in size the complexity of the control problem is increased by the number of modes that must be controlled. However, due to many limitations it is not always possible to control all of the modes of the structure and a reduced order model of the flexible system must be used. But this reduced order model creates new problems since the residual (non-modeled) modes of the structure can be excited by the control inputs causing control spillover. The reduction and elimination of this spillover is of critical importance in designing a precise control system.

In this paper we will examine a new approach forwarded in [2] that is based on the independent modal-space control method. This approach uses a finite number of spatially distributed input points that essentially eliminates control spillover on the modes beyond the control model. We will first give an overview of the formulation of this method, then we will apply the approach through computer simulation to an undamped beam and an undamped truss problem. We will use the results to determine if the approach to the control problem proposed in [2] can reduce or eliminate the control spillover.

CHAPTER I

CONTROL SYSTEM FORMULATION

Introduction

In this chapter we will overview the formulation for the control method presented in [2]. The flexible structure will be examined as a distributed-parameter system. By discretizing this system in space one converts the partial differential equation into an infinite set of ordinary differential equations for the modal coordinates. By using the independent modal-space control method (IMSC) and the integral formulation of the structural dynamic system one can design a desired control system.

The Distributed-Parameter System

The displacement, $u(x,t)$, of an arbitrary point x of an undamped nongyroscopic distributed-parameter system can be expressed by a partial differential equation [3]

$$Lu(x,t)+M(x)\partial^2u(x,t)/\partial t^2=f(x,t) \tag{1}$$

which must be satisfied at every point x of the domain D. In equation (1) L is a self-adjoint, positive definite linear differential operator expressing the system stiffness, $M(x)$ is the mass distribution, and $f(x,t)$ is a

distributed input. In addition the displacement must also satisfy given boundary conditions.

When solving equation (1) it is observed that the system possesses a denumerable infinite set of eigenvalues λ_r, which are related to the natural frequencies by $\lambda_r = \omega_r^2$, $r=1,2,\ldots$, and eigenfunctions $\phi_r(x)$. Because L is self-adjoint the eigenfunctions are orthogonal and can be normalized to satisfy

$$\int_D M\phi_s\phi_r dD = \delta_{rs}, \qquad r,s=1,2,\ldots \tag{2}$$

$$\int_D \phi_s L\phi_r dD = \omega_r^2\delta_{rs}, \qquad r,s=1,2,\ldots \tag{3}$$

in which δ_{rs} is the Kronecker delta.

By using the expansion theorem [4] the solution for the system can be represented by an infinite series of space-dependent eigenfunctions $\phi_r(x)$ multiplied by a time-dependent generalized modal coordinate $q_r(t)$ of the form

$$u(x,t) = \sum_{r=1}^{\infty} \phi_r(x) q_r(t). \tag{4}$$

By placing equation (4) into equation (1), multiplying by $\phi_s(x)$, integrating over D, and using the orthogonality equations (2) and (3) we obtain

$$\ddot{q}_r(t) + \omega_r^2 q_r(t) = f_r(t), \qquad r=1,2,\ldots \tag{5}$$

where

$$f_r(t) = \int_D \phi_r(x) f(x,t) dD, \qquad r=1,2,\ldots \tag{6}$$

are generalized modal control forces. Because it is seldom feasible to control the entire infinity of modes we must reduce the distributed-parameter model to a finite number of modes that are sufficient to accurately model the system.

Independent Modal-Space Control

If the mathematical model of our system is very large it is not always practical and economically feasible to control all of the modelled modes. Therefore, the model is broken into two groups the controlled modes and the uncontrolled or residual modes. By assuming that the control input is implemented by m point actuators, then the actuator forces can be described as discrete by [4]

$$f(x,t) = \sum_{i=1}^{m} F_i(t)\delta(x-x_i), \tag{7}$$

where $\delta(x-x_i)$ is a spatial Dirac delta function. By placing equation (7) into equation (6) we obtain

$$f_r(t) = \sum_{i=1}^{m} \phi_r(x_i)F_i(t), \qquad r=1,2,.... \tag{8}$$

By examining equation (8) one can see that the actual control forces $F_i(t)$ can excite each mode. However, these forces are designed to suppress vibration in the controlled modes, which results in excitation of the uncontrolled modes. This excitation of the uncontrolled modes is referred to as control spillover. This spillover exists

because the actuators are discrete elements. If distributed actuators were available, the modal control forces could be generated so that they do not excite the residual modes and therefore eliminate control spillover.

We can put equations (5) and (8) into compact form by defining the following vectors

$$q(t)=\{q_1(t),\ q_2(t),\ \ldots\}^T, \tag{9}$$

$$f(t)=\{f_1(t),\ f_2(t),\ \ldots\}^T, \tag{10}$$

$$F(t)=\{F_1(t),\ F_2(t),\ \ldots,\ F_m(t)\}^T, \tag{11}$$

and the matrices

$$\Omega^2=\mathrm{diag}(\omega_1{}^2,\ \omega_2{}^2,\ \ldots), \tag{12}$$

$$B=[\phi_r(x_s)], \qquad r=1,2,\ldots,\quad s=1,2,\ldots m. \tag{13}$$

Then equation (5) can be expressed as

$$\ddot{q}(t)+\Omega^2 q(t)=f(t) \tag{14}$$

and equation (8) becomes

$$f(t)=BF(t). \tag{15}$$

From equation (15) it can be seen that the actual control force can be obtained from the modal control force by

$$F(t)=B^{-1}f(t). \tag{16}$$

The modal control forces in general can be designed as desired in the form of $f_r(q,\dot{q})$ to design a closed-loop control system. However, if each modal force f_r is a feedback function of all modal coordinates q, then the modal responses become coupled during control, so that modal coordinates q_r no longer represent natural

coordinates for the controlled system. On the other hand, if each modal input is designed as a function in the form of feedback of only its own modal coordinates, i.e. $f_r(q_r, \dot{q}_r)$ then the response of each controlled mode will be independent of the responses of other modes and each q_r will retain its natural property. This particular technique of modal feedback control is known as Independent Modal-Space Control [5].

Integral Formulation of Structural Dynamic Systems

The structural dynamic equation of a self-adjoint, positive definite distributed-parameter system can also be written alternately in the integral form [6]

$$\bar{u}(x,t) = u(x,t) + \int m(\eta) C(x,\eta) \ddot{u}(\eta,t) d\eta = \int C(x,\eta) f(\eta,t) d\eta, \qquad (17)$$

where x, η denote the structural domain. $\bar{u}(x,t)$ is the instantaneous static deflection caused by $f(\eta,t)$. $C(x,\eta)$ is the flexibility influence function that is dependent on the boundary conditions, material properties, and geometry of the system.

It can be shown [2] that if we express the spatial input distribution $f(\eta,t)$ and the flexibility function $C(x,\eta)$ in the form

$$f(\eta,t) = \sum_{r=1}^{n} m(\eta) \phi_r(\eta) f_r(t), \qquad (18)$$

$$C(x,\eta) = \sum_{r=1}^{\infty} \phi_r(x)\phi_r(\eta)/\omega_r^2, \tag{19}$$

where ϕ_r are eigenfunctions and $f_r(t)$ are modal input forces, then the instantaneous static displacement will have the form

$$\bar{u}(x,t) = \sum_{r=1}^{n} \phi_r(x)\omega_r^{-2} f_r(t), \qquad r=1,2,\ldots n. \tag{20}$$

If we introduce the vector

$$b_N^T = \{\phi_1(x) \ \ldots \ \phi_r(x) \ \ldots \ \phi_n(x)\} \tag{21}$$

then equation (20) can be written as

$$\bar{u}(x,t) = b_N^T \Omega_N^{-2} f(t), \tag{22}$$

where

$$f(t) = \{f_1 \ f_2 \ \ldots \ f_r \ \ldots \ f_n\}. \tag{23}$$

If we now look at equation (17) and consider $\bar{u}(x,t)$ at a finite number of points with the load $f(x,t)$ at a finite number of input points and using the collocation technique [2] so that the controlled points are the same as the input locations, $x_m = \eta_m$, then we can rewrite equation (17) as

$$\begin{bmatrix} \bar{u}_1 \\ \vdots \\ \bar{u}_m \\ \hline \bar{u}_{m+1} \\ \vdots \\ \bar{u}_\infty \end{bmatrix} = \left[\begin{array}{cccc|ccc} c_{11} & c_{12} & \cdots & c_{1m} & c_{1m+1} & \cdots & c_{1\infty} \\ c_{m1} & \cdots & \cdots & c_{mm} & c_{mm+1} & \cdots & c_{m\infty} \\ \hline c_{m+11} & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ c_{\infty 1} & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \end{array}\right] \begin{bmatrix} F_1 \\ \vdots \\ F_m \\ \hline 0 \\ \vdots \\ 0 \end{bmatrix}. \tag{24}$$

In equation (24)

$$c_{ij} = C(x_i, x_j), \qquad i,j = 1,2,\ldots \tag{25}$$

$$\bar{u}_i = \bar{u}(x_i, t), \qquad i=1,2\ldots \tag{26}$$

$$f(x_K, t) = F_K(t)\,\delta(x - x_K) \qquad K=1,2,\ldots m \tag{27}$$

and it can be easily rearranged as

$$\begin{bmatrix} \bar{u}_1^* \\ \vdots \\ \bar{u}_m^* \\ \hline \bar{u}_{m+1} \\ \vdots \\ \bar{u}_\infty \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} & \cdots & c_{1m} \\ c_{m1} & \cdots & \cdots & c_{mm} \\ \hline c_{m+11} & \cdots & \cdots & \cdots \\ c_{\infty 1} & \cdots & \cdots & \cdots \end{bmatrix} \begin{bmatrix} F_1 \\ \vdots \\ F_m \end{bmatrix} = \begin{bmatrix} C \\ - \\ C_R \end{bmatrix} F \tag{28}$$

where $*$ denotes a desired (prescribed) behavior. Hence $\bar{u}^*(x_i) = \bar{u}_i^*$, $i=1,2,\ldots m$, is the desired instantaneous static displacement for the controlled points x_i, $i=1,2,\ldots m$, which can be realized by designing the input forces F. The C matrix contains all the flexibility coefficients for the controlled points and the C_R matrix contains the coefficients for the residual points. From equation (28) we can easily find the unique solution

$$F(t) = C^{-1}\bar{u}^*, \tag{29}$$

where

$$\bar{u}^* = \{\bar{u}_1^* \; \bar{u}_2^* \; \ldots \; \bar{u}_m^*\}^T. \tag{30}$$

Specified Response Model

In view of equation (29) we need to devise a method by which the desired instantaneous static response vector u^* can be specified. From equation (4) it is easy to show that a desired displacement profile $u^*(x,t)$ can be express-

ed in a similar form, but by using a desired modal coordinate q^*

$$u^*(x,t)=\sum_{r=1}^{n}\phi_r(x)q_r^*(t). \tag{31}$$

It then follows that we can state a set of second order model dynamics for the coordinates q_r^* as

$$\ddot{q}_r^*+\Omega_r^2q_r^*=f_r^*(t), \qquad r=1,2,\ldots n, \tag{32}$$

where f_r^* is a model forcing function. It can be seen that by adjusting the model coordinates q_r^* in equation (31) one can generate a desired displacement $u^*(x,t)$. This can be accomplished by designing the model inputs in the form $f_r^*(q_r^*,\dot{q}_r^*)$ such that q_r^* behaves as desired.

By using the same method as used in producing equation (22) it can now be shown that

$$\bar{u}^*(x,t)=b_N^T\Omega_N^{-2}f_N^*(t). \tag{33}$$

If we now evaluate equation (33) at a finite number of m points, $\bar{u}^*(x_i)=\bar{u}_i^*$ $i=1,2,\ldots m$, and form the vector $\bar{u}^*$ we obtain

$$\bar{u}^*=B_N^T\Omega_N^{-2}f^*(t) \tag{34}$$

$$B_N=[b_N(x_1)\ b_N(x_2)\ \ldots\ b_N(x_m)] \tag{35}$$

$$b_N(x_K)=\{\phi_1(x_K)\ \phi_2(x_K)\ \ldots\ \phi_r(x_K)\ \ldots\phi_n(x_K)\}^T \tag{36}$$

If we now substitute equation (34) into equation (29) we obtain

$$F(t)=C^{-1}\bar{u}^*=C^{-1}B_N^T\Omega_N^{-2}f_N^*(t) \tag{37}$$

Where $f_N^*(t)$ is an n-vector of model inputs $f_N^*(t)=\{f_1^*\ f_2^*\ \ldots\ f_n^*\}^T$ designed by independent modal-space control

of the model given by equation (32). This is equivalent to designing a model following input force F for the actual structure in an open-loop sense [2].

Open Loop Control

If the input forces F in equation (15) are designed on the basis of model inputs f^*, equation (37), of a (modal) reference model, equation (32), and are applied directly to the structure through equation (14) without regard to the actual response of the structure, then the response design is essentially an open-loop control system. By introducing equation (37) into equation (14), which is the structural modal set, we obtain

$$\ddot{q}(t)+\Omega^2 q(t)=B_S C^{-1} B_N^T \Omega_N^{-2} f_N^*(t), \tag{38}$$

where

$$B_S=[B_N^T \;\vdots\; B_R^T]^T, \tag{39}$$

$$B_R=[b_R(x_1) \;\ldots\; b_R(x_m)], \tag{40}$$

$$b_R(x_K)=\{\phi_{n+1}(x_K) \;\; \phi_{n+2}(x_K) \;\ldots\}^T, \tag{41}$$

and

$$q=\{q_N^T \;\; q_R^T\}^T, \tag{42}$$

$$q_N=\{q_1 \;\; q_2 \;\ldots\; q_n\}^T, \;\; q_R=\{q_{n+1} \;\; q_{n+2} \;\ldots\}^T. \tag{43}$$

If we expand equation (38) in terms of B_N and B_R we get

$$\ddot{q}(t)+\Omega^2 q(t)=[B_N^T \;\; B_R^T]^T C^{-1} B_N^T \Omega_N^{-2} f_N^*(t). \tag{44}$$

One can easily see that there will be control spillover to the residual dynamics $q_R(t)$ from the model input f^*. However, if the structure had a finite number, N, of modes

and $B_S = B_N$ then there would be no residual modes to incur spillover.

Another approach to eliminate spillover to residual modes $r = n+1,\ n+2, \ldots$ if the system has an infinite number of modes, is to increase the number of inputs $m \rightarrow \infty$. It can be shown [7, 2] that the triple matrix product $B_N C^{-1} B_N^T$ will diagonalize to the eigenvalues matrix Ω_N^2. This can be seen by remembering that the eigenvalue problem of a self-adjoint distributed-parameter system has the orthogonality property

$$\int \phi_s(x) L \phi_r(x)\, dD = \omega_r^2 \delta_{rs}, \tag{45}$$

where L is the distributed stiffness operator. By recalling the form of B_N and B_R, noting that the C^{-1} matrix is the stiffness matrix K and considering the distributed orthogonality, one can see that the triple matrix product $B_N C^{-1} B_N^T$ is the discrete approximation of equation (45). Therefore as the number of inputs goes to infinity the system starts to resemble the distributed-parameter system more closely which results in

$$B_N C^{-1} B_N^T = B_N K B_N^T \longrightarrow [\Omega_N^2] \tag{46}$$

$$B_R C^{-1} B_N^T = B_R K B_N^T \longrightarrow 0. \tag{47}$$

If we apply this to equation (44) we get

$$\ddot{q}(t) + \Omega^2 q(t) = [I \,\vdots\, 0]^T f^*(t) \tag{48}$$

where [I] is the NxN identity matrix. This is essentially the same as equation (14). Since the modal inputs $f^*(t)$

are designed in the form $f^*(q^*,\dot{q}^*)$, provided that the initial conditions on the model and the structure are the same, the open-loop approach for modal model following is equivalent to a modal feedback control law on the actual structure since one will have $q=q^*$ $t\geq 0$.

Closed-Loop Control

For the closed-loop case the input forces $f^*(t)$ are designed in the form $f^*(q_N,\dot{q}_N)$ and we let

$$f_N{}^*(q_N,\dot{q}_N)=H_1q_N+H_2\dot{q}_N, \qquad (49)$$

where H_1 and H_2 are NxN modal control gains of the form

$$H_1=\beta[I], \quad H_2=\alpha[I]. \qquad (50)$$

If we apply this to the model n-modes of equation (38) we get the form

$$\ddot{q}_N+\Omega_N{}^2q_N=\beta B_NC^{-1}B_N{}^T\Omega_N{}^{-2}q_N+\alpha B_NC^{-1}B_N{}^T\Omega_N{}^{-2}\dot{q}_N. \qquad (51)$$

We can rearrange this equation and we get the form

$$\ddot{q}_N-\alpha B_NC^{-1}B_N{}^T\Omega_N{}^{-2}\dot{q}_N+(\Omega_N{}^2-\beta B_NC^{-1}B_N{}^T\Omega_N{}^{-2})q_N=0. \qquad (52)$$

It can be easily seen that the residual dynamics will have the form

$$\ddot{q}_R+\Omega_R{}^2q_R=\alpha B_RC^{-1}B_N{}^T\Omega_N{}^{-2}\dot{q}_N+\beta B_RC^{-1}B_N{}^T\Omega_N{}^{-2}q_N. \qquad (53)$$

If we examine equations (52) and (53) as the number of inputs is increased to infinity it can easily be seen, by virtue of equations (46) and (47), that the control model, equation (52), goes to the ideal case where the controlled distributed-parameter system behaves as

$$\ddot{q}_N-\alpha\dot{q}_N+(\Omega_N{}^2-\beta)q_N=0, \qquad (54)$$

and the residual excitation totally disappears. Essentially, the controlled behavior exhibited by equation (54) is precisely the behavior that will be obtained if a spatially continuous input as in equation (18) is used with

$$f_r = \alpha\dot{q}_r + \beta q_r \qquad r = 1, 2, \ldots n. \tag{55}$$

By choosing α as a real negative number and β as a real number less than ω_r^2, equation (54) represents a stable system where α can be regarded as a modal damping coefficient and β as a modal stiffness coefficient. With the elimination of the residual modes the n controlled modes will be uniformly damped and stiffened.

We have seen that the feedback control of the structure is performed via an input control force in the form

$$F(t) = C^{-1}B_N{}^T\Omega_N{}^{-2}f_N(t). \tag{56}$$

It can be shown [2] that the spatially discrete inputs F are redistributed by synthesizing the modal feedback quantity $f_N(t)$ through the C matrix so as to create effective continuously distributed damping and stiffness inputs (see [2] for proof). Therefore equation (56) is referred to as "spatial modal input-distribution control" [2] that has an inherent spatial orthogonal non-pass filtering property with respect to the residual modes. This occurs through the gain factor $C^{-1}B_N{}^T$ that eliminates spillover as the number of inputs increase. However, the number of inputs for most structures does not have to go to infinity before the model essentially reaches the ideal

case and the residual effect is small enough to be neglected.

CHAPTER II

CONTROL OF AN UNDAMPED BEAM

Introduction

In this chapter the closed-loop feedback control technique developed in the last chapter will be applied to a simply supported beam. The method of implementing the control system will be examined, then three examples of the control system will be shown.

Implementation

To implement the closed-loop feedback control method we must put the modal equations into state form. The modal equation for a distributed-parameter system, equation (5), can easily be put into state form by using the auxiliary variable $\xi_r(t)$ defined by

$$\dot{q}_r(t)=\omega_r\xi_r(t), \qquad r=1,2,\ldots. \tag{57}$$

The resulting equation in modal state form is found to be [8]

$$\dot{w}_r(t)=\Lambda_r w_r(t)+W_r(t), \tag{58}$$

where $w_r(t)$ is the modal state vector, $W_r(t)$ the associated modal control vector, and Λ_r the coefficient matrix defined by

$$w_r(t)=[q_r(t)\ \ \varepsilon_r(t)]^T, \tag{59}$$

$$W_r(t)=[0\ \ f_r(t)/\omega_r]^T, \tag{60}$$

$$\Lambda_r=\begin{bmatrix} 0 & \omega_r \\ -\omega_r & 0 \end{bmatrix}, \qquad r=1,2,\ldots \tag{61}$$

However, the independent modal-space control method designs the control vector in the form

$$W_r(t)=G_r w_r(t), \tag{62}$$

where G_r is a 2x2 modal control gain matrix. The ideal modal gain can be found by putting equation (54) into state form since it represents the distributed-parameter system behavior for the ideal control model. The resulting control design model has the form

$$\dot{w}_r(t)=\Lambda_r w_r(t)+G_r w_r(t), \qquad r=1,2,\ldots n \tag{63}$$

where the ideal gain matrix is defined by

$$G_r=\begin{bmatrix} 0 & 0 \\ \beta/\omega_r & \alpha \end{bmatrix} \qquad r=1,2,\ldots n. \tag{64}$$

The ideal closed-loop eigenvalues can be found from equation (52) and will have the form

$$\lambda_r=\alpha/2 \pm \sqrt{\alpha^2-4\omega_r^2-4\beta}/2. \tag{65}$$

To implement the control design the continuous-time modal state equations must be discretized by sampling in time to permit digital state estimation and control. This can be shown [9] to result in the form

$$w_r(k+1)=\Phi_r w_r(k)+\Gamma_r W_r(k), \qquad k=1,2,\ldots, \quad r=1,2\ldots n \tag{66}$$

where

$$\Phi_r(\tau) = \begin{bmatrix} \cos\omega_r\tau & \sin\omega_r\tau \\ -\sin\omega_r\tau & \cos\omega_r\tau \end{bmatrix}, \tag{67}$$

$$\Gamma_r(\tau) = (1/\omega_r) \begin{bmatrix} \sin\omega_r\tau & 1-\cos\omega_r\tau \\ -(1-\cos\omega_r\tau) & \sin\omega_r\tau \end{bmatrix}, \tag{68}$$

in which τ denotes the sampling time and the arguments $k+1$ and k denote the times $t=(k+1)\tau$ and $t=k\tau$ respectively.

The simply supported beam (Fig. 1) is chosen with unit length, unit mass distribution, and unit stiffness. The eigenfunctions and natural frequencies of the beam are known to be [10]

$$\phi_r = \sqrt{2/mL}\ \sin r\pi x/L, \tag{69}$$

$$\omega_r = r^2\pi^2 \sqrt{EI/mL^4}, \qquad r=1,2,\ldots \tag{70}$$

where m is the mass, L is the length, and EI is the stiffness. The closed form of the influence function $C(x,\eta)$ for the beam with K inputs can be found to be [2,9]

$$C(x_i,\eta_i) = x_i/6EI\ (x_i^2\eta_i/L+\eta_i^3/L+2\eta_i L-x_i^2-3\eta_i^2), \quad x_i \le \eta_i \tag{71}$$

for $i=1,2,\ldots K$. If $x_i \ge \eta_i$, then the η and x terms are interchanged.

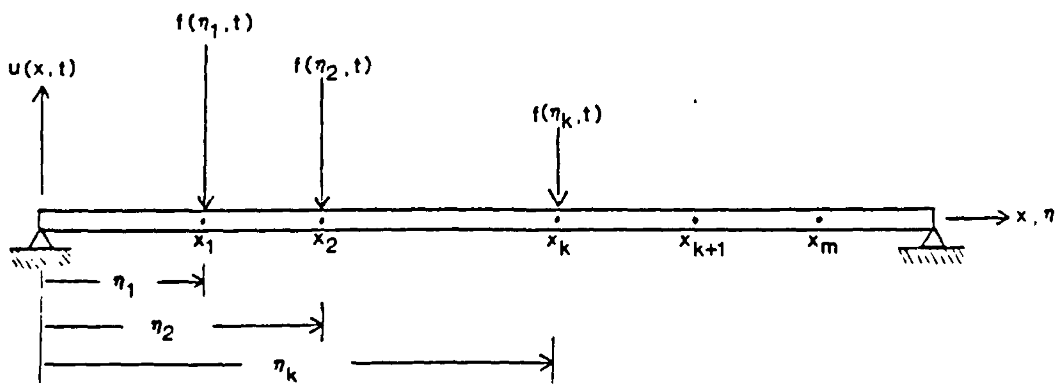


Fig. 1. Simply supported uniform beam

Computer simulation of the control system is accomplished by the use of the computer program PNTCNT (Appendix A). The program output consists of plots of the displacement response of a chosen control point versus time for the reduced order model and the structure which includes the residual modes. The input force at the same point versus time is also plotted. The horizontal axis on all plots reflect time and the vertical axis reflect either displacement or the input force. Since a beam has an infinite number of modes the structure is modelled by a finite number, N_s, of modes that are sufficient to accurately represent the infinite set of modes. The reduced order model that is controlled consists of $N=n$ modes where $N<N_s$ and the number of inputs will be referred to as K. The input points are picked arbitrarily along the beam, but no points are allowed to coincide with any of the structures nodes since that would reduce their effectiveness. The initial modal state disturbances for all the examples are chosen as $q_r(0)=0$ and $\dot{q}_r(0)=1.0$ units/second, $r=1,2,...N$.

Example 1

In this first example the beam has unit length, $L=1$, and the structure is represented by ten modes, $N_s=10$. The reduced order model consists of the first four modes of the structure, $N=4$. The damping coefficient is chosen as $\alpha=-40$ and the stiffness coefficient is chosen as $\beta=0$. The

point to observe the model and structure responses at was chosen as $x=0.21$ units. The time tics on the plots are at 0.5 second intervals and the displacement and force tics are at 1.0 unit intervals. The displacement response plots for both the model and the structure can be seen in Figures 2-11. When $K=2$ the input points are at $x=0.09$ and $x=0.21$. When $K=4$ the points $x=0.31$, 0.42 are added as input points. When $K=6$ the points $x=0.53$, 0.64 are added, for $K=8$ points $x=0.73$, 0.84 are added and for $K=9$ the point $x=0.93$ is added as an input point.

As we examine Figures 2-11 it can be seen that as the number of inputs increase the model and the structure responses start looking more and more alike until they match each other when $K=9$. If we compare Figures 10 and 11 to Figure 12, which is the response of the ideal model, equation (54), that is based on an infinite number of input points, i.e. on a continuously distributed input we see that they are identical. We can confirm that the actual model approaches the ideal model by comparing the gain matrices and the closed-loop eigenvalues of the actual model to the ideal gain matrix and eigenvalues. The ideal gain matrix has the form

$$G=\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -40 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -40 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -40 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -40 \end{bmatrix}. \tag{72}$$

When K=2 the non-ideal gain matrix is

$$G_n = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -34.37 & 0 & -12.14 & 0 & -3.48 & 0 & .91 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -12.14 & 0 & -7.03 & 0 & -5.49 & 0 & -4.62 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -3.48 & 0 & -5.49 & 0 & -6.98 & 0 & -7.60 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & .91 & 0 & -4.62 & 0 & -7.60 & 0 & -8.95 \end{bmatrix}. \quad (73)$$

When K=6 the non-ideal gain matrix is

$$G_n = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -39.92 & 0 & -.45 & 0 & -.93 & 0 & -.97 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -.45 & 0 & -37.22 & 0 & -5.72 & 0 & 6.13 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -.93 & 0 & -5.72 & 0 & -27.82 & 0 & -13.48 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -.97 & 0 & 6.13 & 0 & -13.48 & 0 & -23.11 \end{bmatrix}. \quad (74)$$

When K=9 the non-ideal gain matrix is

$$G_n = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -40.00 & 0 & -.0002 & 0 & .0004 & 0 & .002 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -.0002 & 0 & -40.00 & 0 & .004 & 0 & .0001 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & .0004 & 0 & .004 & 0 & -39.92 & 0 & .03 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & .002 & 0 & .0001 & 0 & .03 & 0 & -39.66 \end{bmatrix}. \quad (75)$$

Table 1. Example 1 closed-loop eigenvalues

IDEAL	K=2	K=6	K=9
-2.60	-3.08	-2.61	-2.60
-37.40	-33.56	-37.32	-37.39
-20.0±i34.04	-2.5±i38.35	-18.7±i34.89	-20.0±i34.04
-20.0±i86.55	-3.4±i88.65	-13.9±i88.00	-20.0±i86.55
-20.0±i156.6	-4.5±i157.5	-11.4±i156.5	-19.8±i156.7

By examining these matrices and Table 1 it becomes obvious that as the number of inputs increase the actual model does approach the ideal model and the effect of the residual dynamics on the structural response is practically eliminated. Another item to note is the input force. In Figures 14-17 it can be seen that as the number of inputs increase the input force decreases. This is logical since the amount of force required to control the system is being distributed to more input points and the force response eventually matches the ideal, Figure 13.

One more point of interest would be to see the effect on the model and the structure response for $K=9$ if the number of sufficient modes to represent the structure is increased. In Figures 18 and 19 the structure is increased from $N_S=10$ to $N_S=13$ and the effect is negligible.

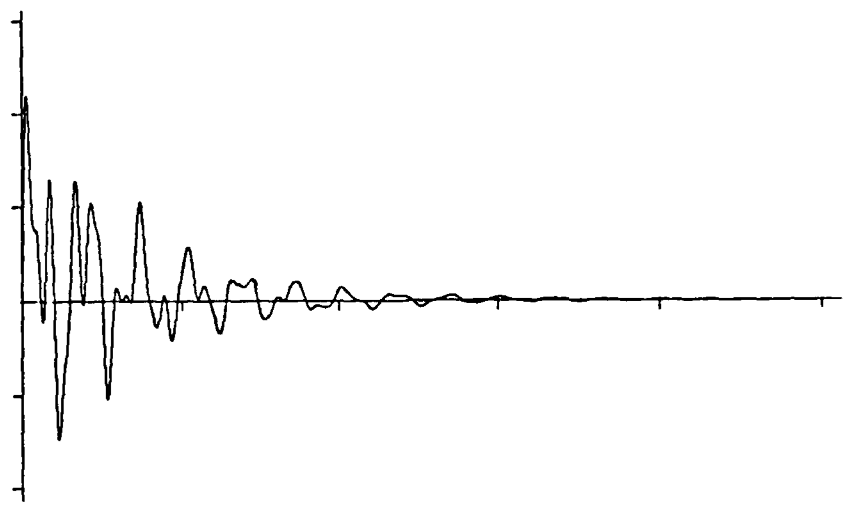


Fig. 2. Example 1 model displacement/time K=2

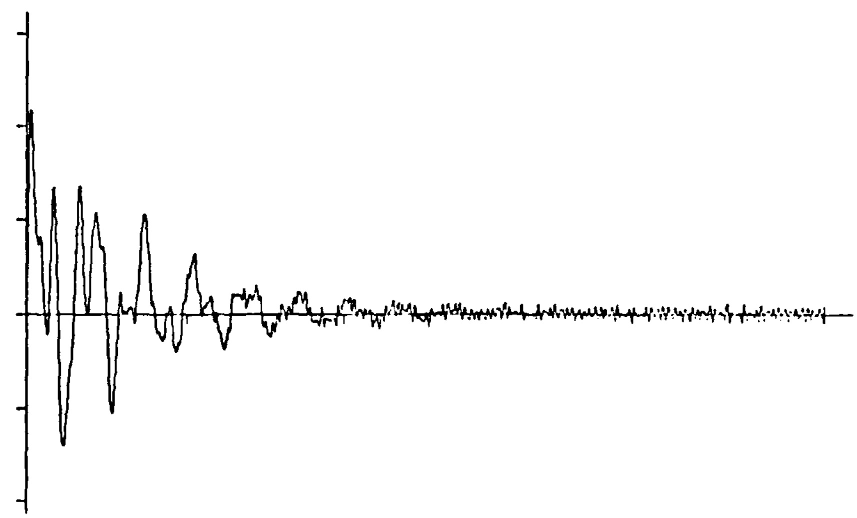


Fig. 3. Example 1 structure displacement/time K=2

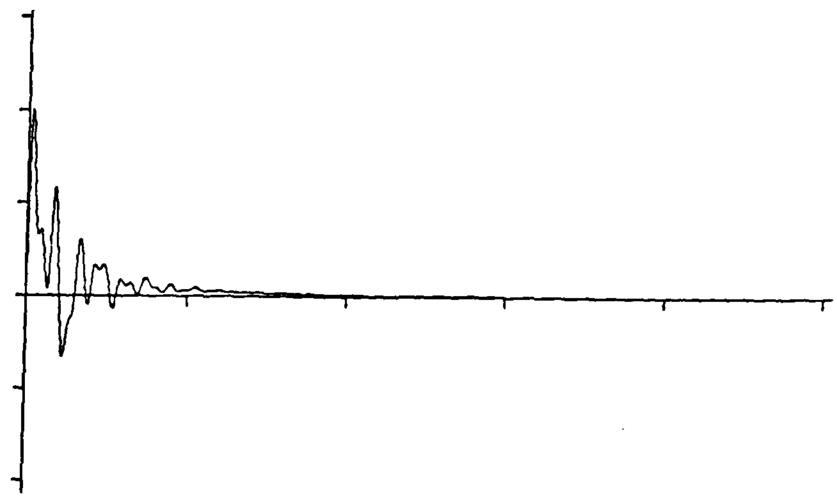


Fig. 4. Example 1 model displacement/time K=4

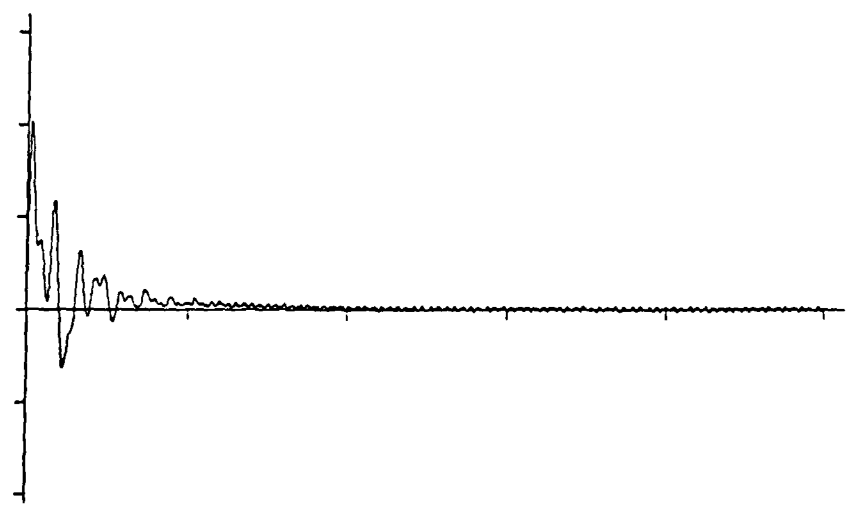


Fig. 5. Example 1 structure displacement/time K=4

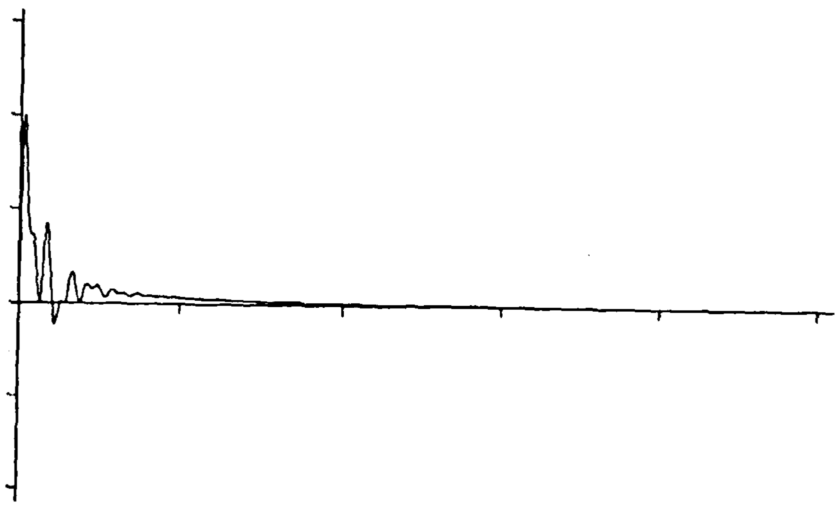


Fig. 6. Example 1 model displacement/time K=6

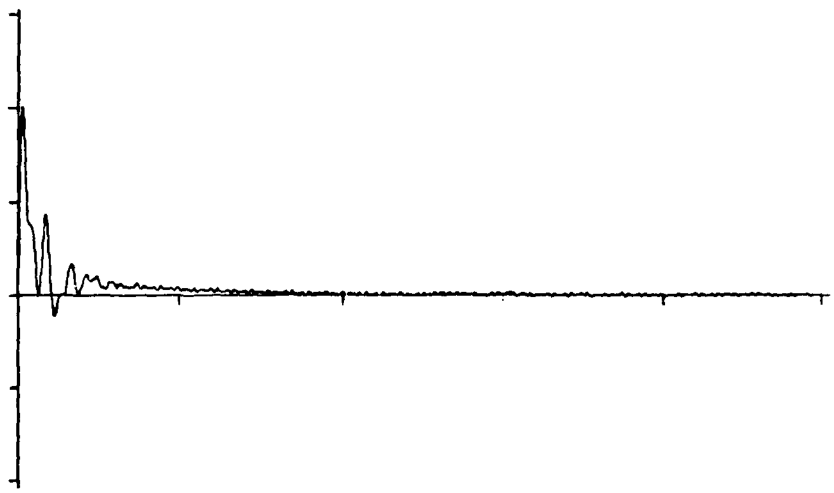


Fig. 7. Example 1 structure displacement/time K=6

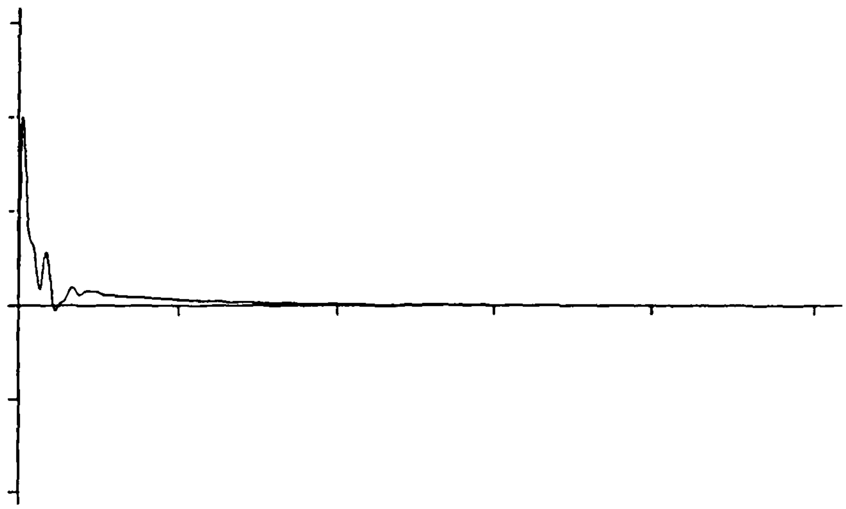


Fig. 8. Example 1 model displacement/time K=8

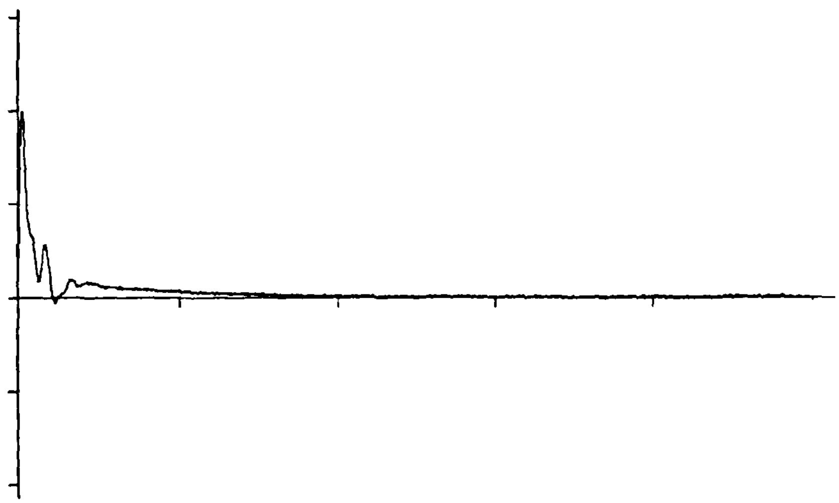


Fig. 9. Example 1 structure displacement/time K=8

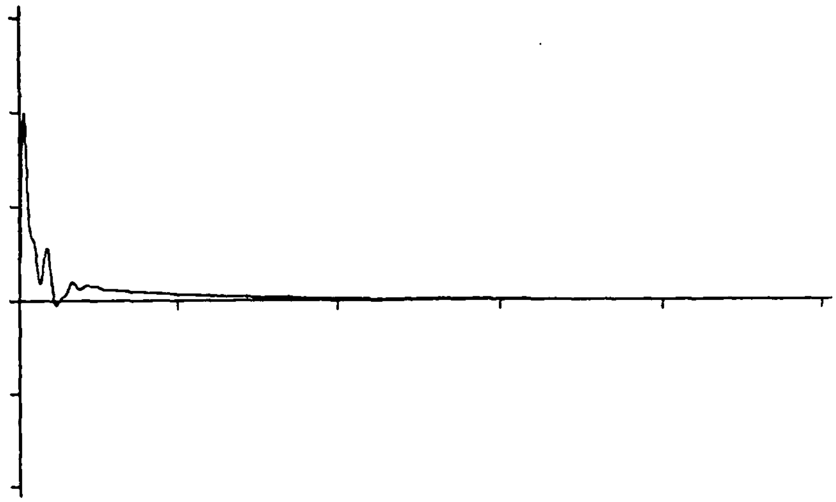


Fig. 10. Example 1 model displacement/time K=9

Fig. 11. Example 1 structure displacement/time K=9

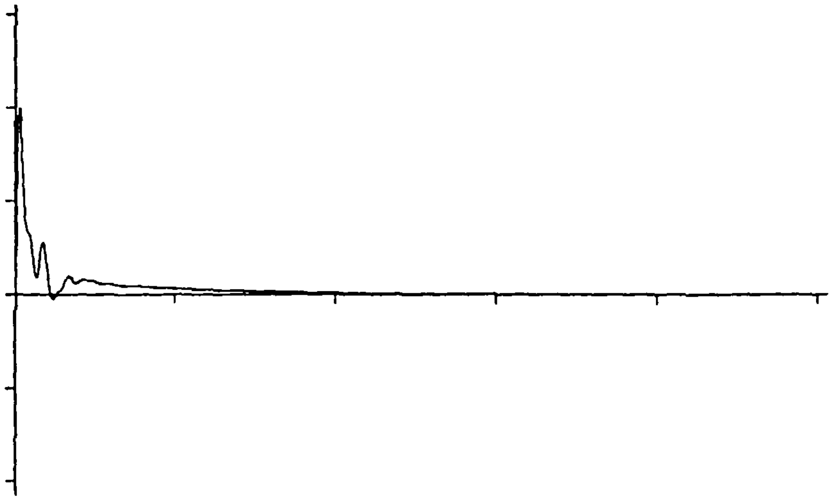


Fig. 12. Example 1 ideal model displacement/time

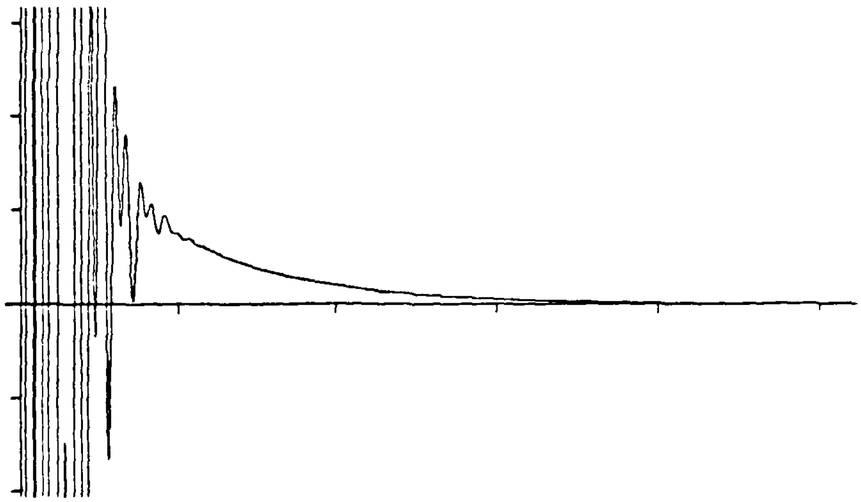


Fig. 13. Example 1 ideal continuous input force/time observed at x=0.21

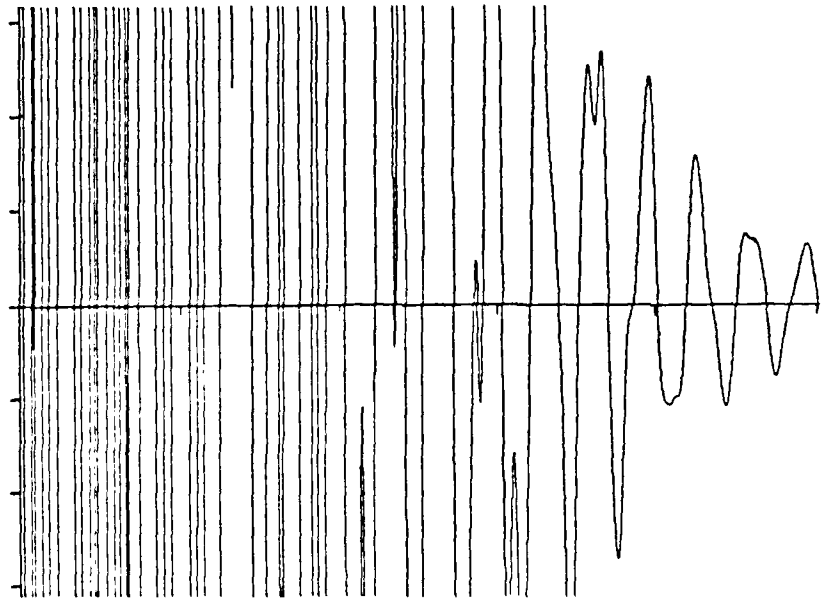


Fig. 14. Example 1 Input force/time K=2

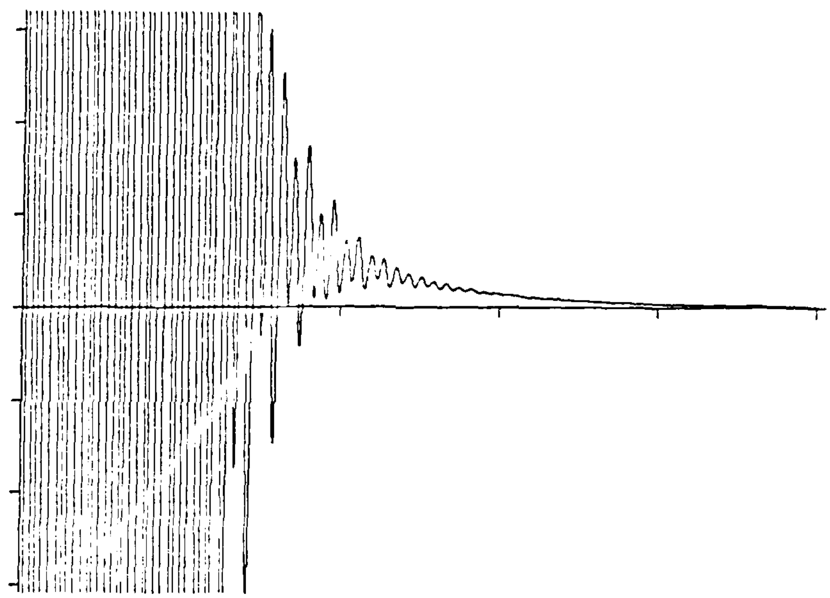


Fig. 15. Example 1 input force/time K=4

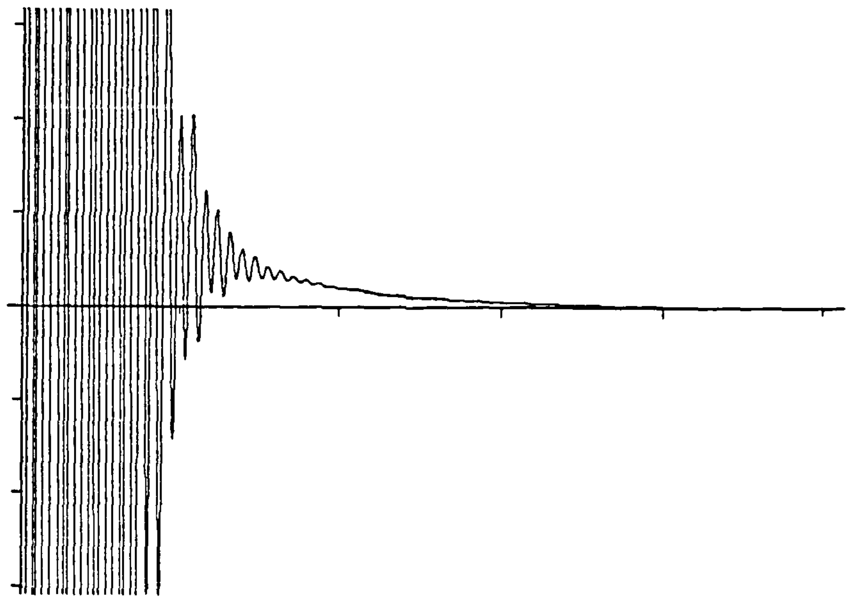


Fig. 16. Example 1 input force/time K=6

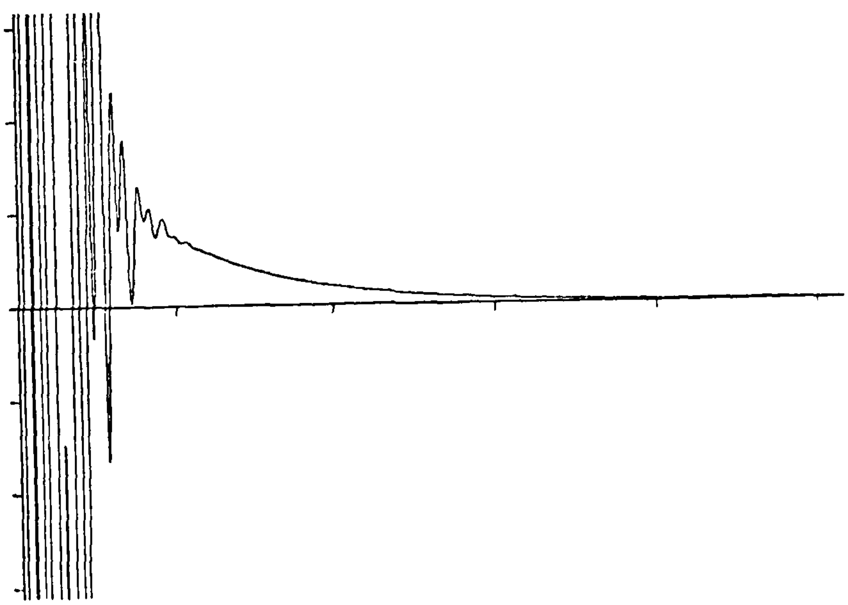


Fig. 17. Example 1 input force/time K=9

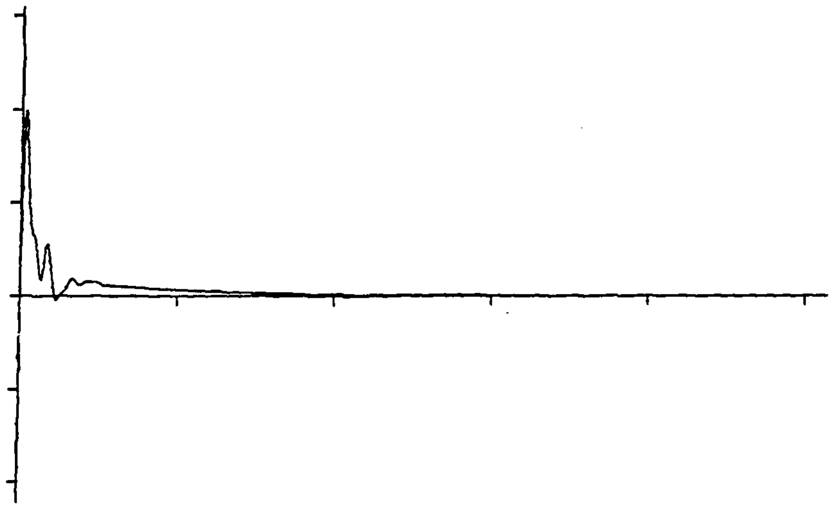


Fig. 18. Example 1 model displacement/time
$K=9$, $N_S=13$

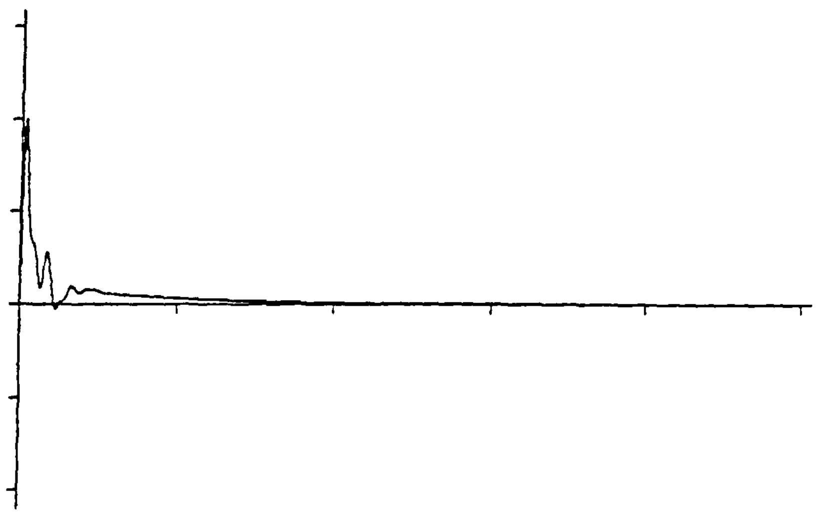


Fig. 19. Example 1 structure displacement/time
$K=9$, $N_S=13$

Example 2

This closed-loop example is very similar to Example 1 since $L=1$, $N_s=10$, $\beta=0$, and the observed point will remain as $x=0.21$. However, we will increase the reduced order model to $N=6$ and the damping coefficient is increased to $\alpha=-80$. The time tics and the displacement and force tics will remain the same as in Example 1. All the input points will also remain the same as in Example 1. The displacement response for the model and the structure can be seen in Figures 20-29. As in the last example both the model response and the structure response, that includes the residuals, move towards the ideal model as the number of inputs increase. At $K=9$ the structure response is again nearly identical to the response of the ideal model.

Table 2. Example 2 closed-loop eigenvalues

IDEAL	K=2	K=9
-1.23	-1.44	-1.23
-78.76	-75.29	-78.76
-33.56	-3.43-i37.79	-33.64
-46.43	-3.43+i37.79	-46.32
-40.00±i79.31	-6.76±i88.57	-39.92±i79.35
-40.00±i152.76	-9.11±i157.14	-39.66±i152.85
-40.00±i243.48	-8.89±i245.23	-39.11±i243.62
-40.00±i353.05	-6.63±i353.83	-38.75±i353.19

If we examine Table 2 we can compare the closed-loop eigenvalues of the ideal model to that of the actual model at K=2 and at K=9. From this table we can confirm that the actual model is almost at the ideal case when K=9. The input force response for K=2-9 can be seen in Figures 30-33 and we can see that as the number of inputs increase the input force decreases towards the ideal distributed case.

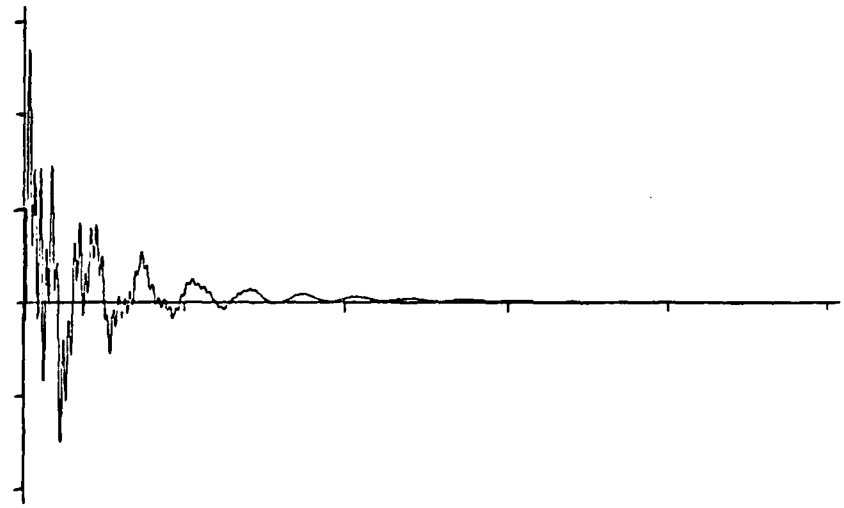


Fig. 20. Example 2 model displacement/time K=2

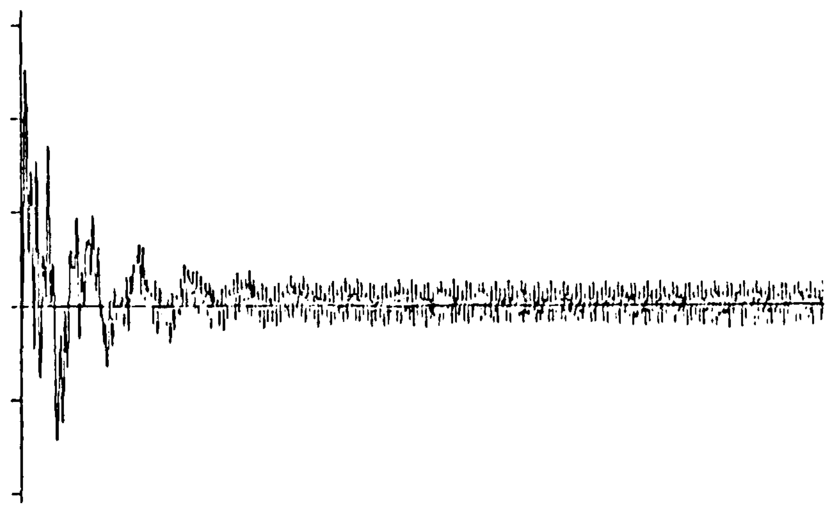


Fig. 21. Example 2 structure displacement/time K=2

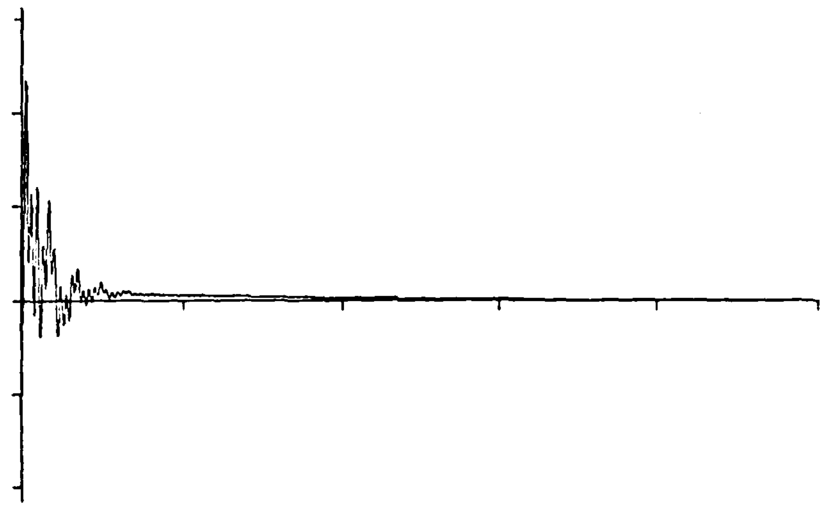


Fig. 22. Example 2 model displacement/time K=4

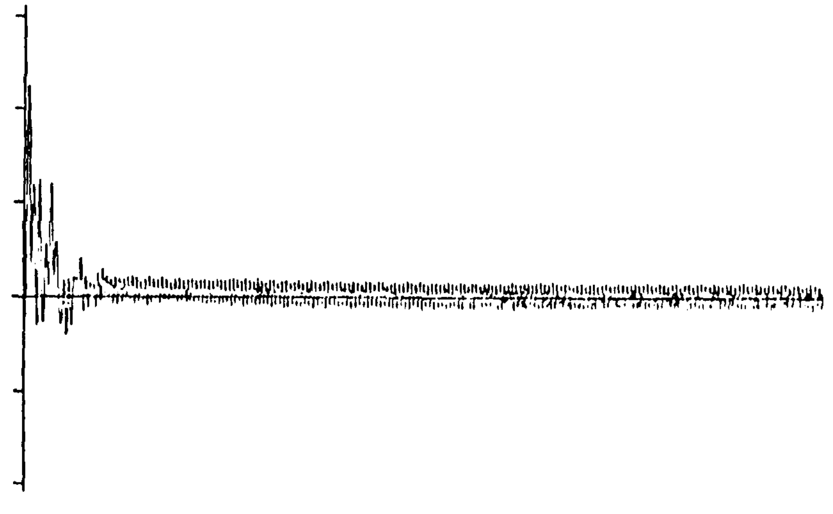


Fig. 23. Example 2 structure displacement/time K=4

Fig. 24. Example 2 model displacement/time K=6

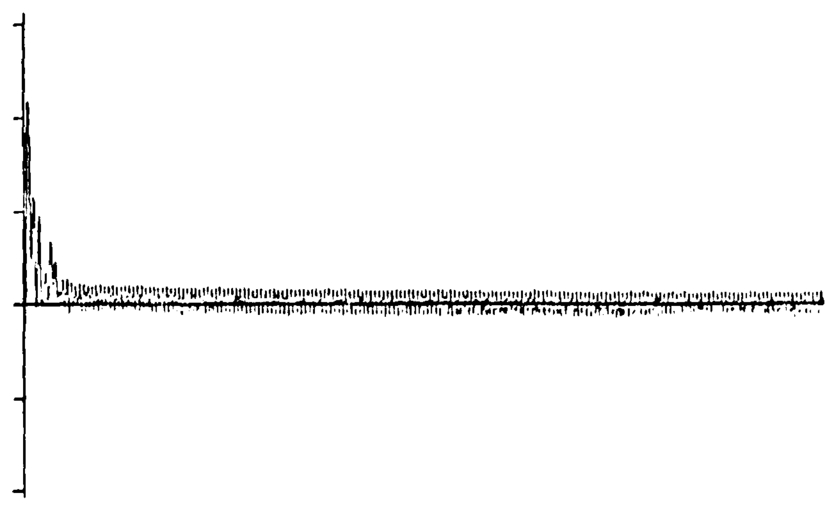


Fig. 25. Example 2 structure displacement/time K=6

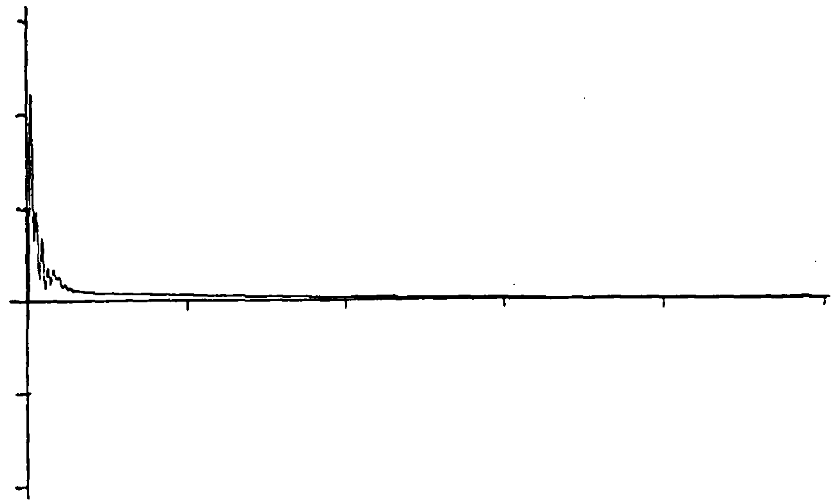


Fig. 26. Example 2 model displacement/time K:8

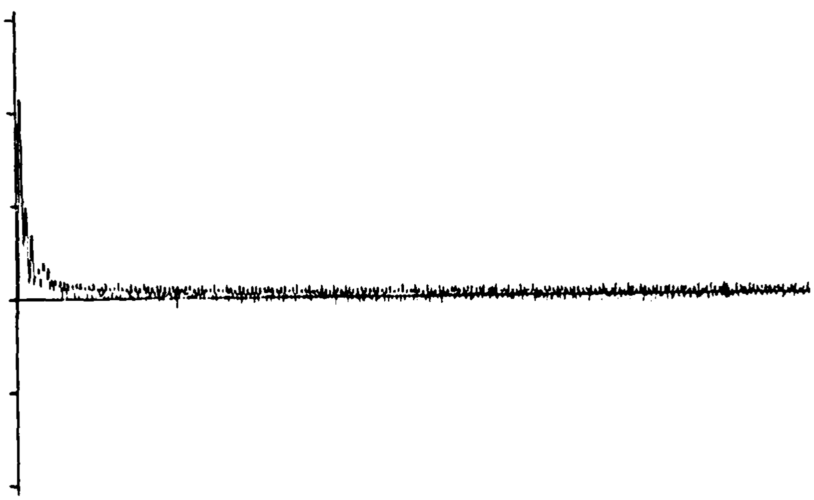


Fig. 27. Example 2 structure displacement/time K:8

Fig. 28. Example 2 model displacement/time K=9

Fig. 29. Example 2 structure displacement/time K=9

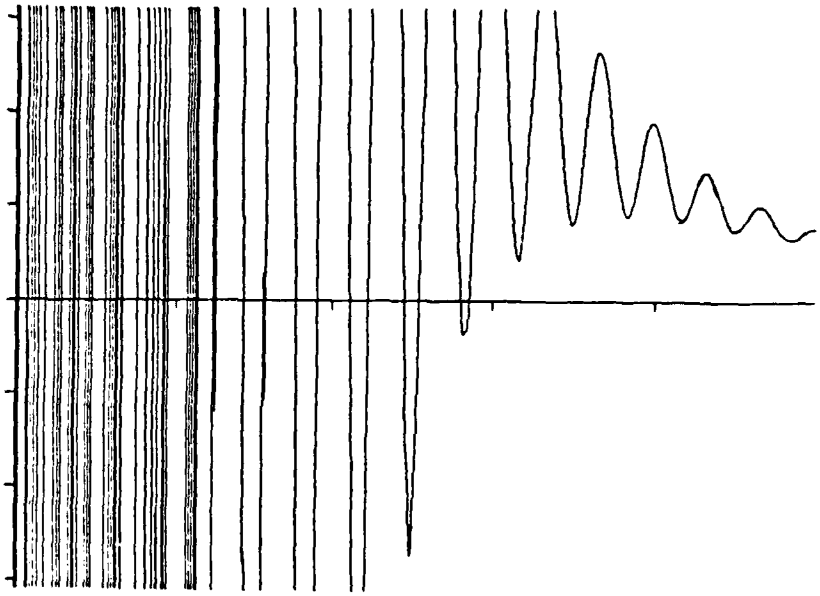


Fig. 30. Example 2 Input force/time K=2

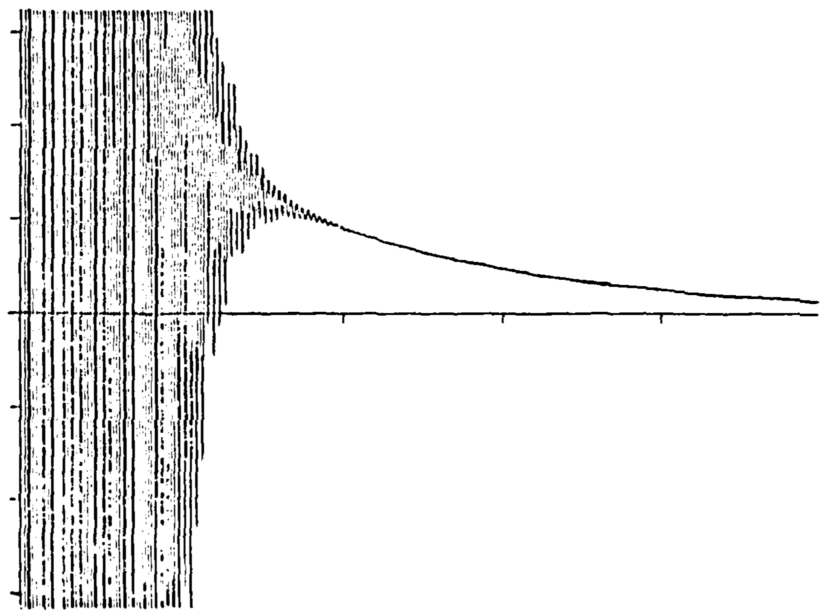


Fig. 31. Example 2 input force/time K=4

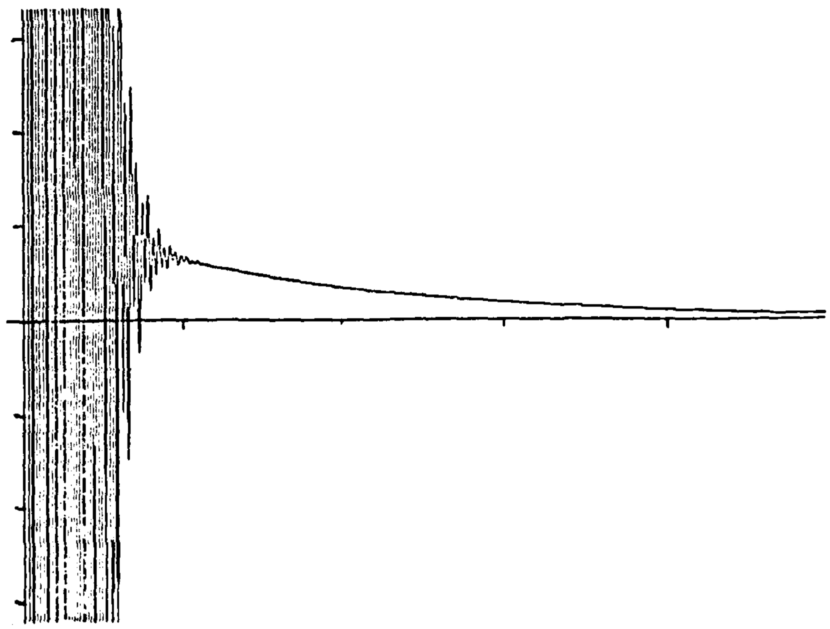


Fig. 32. Example 2 input force/time K=6

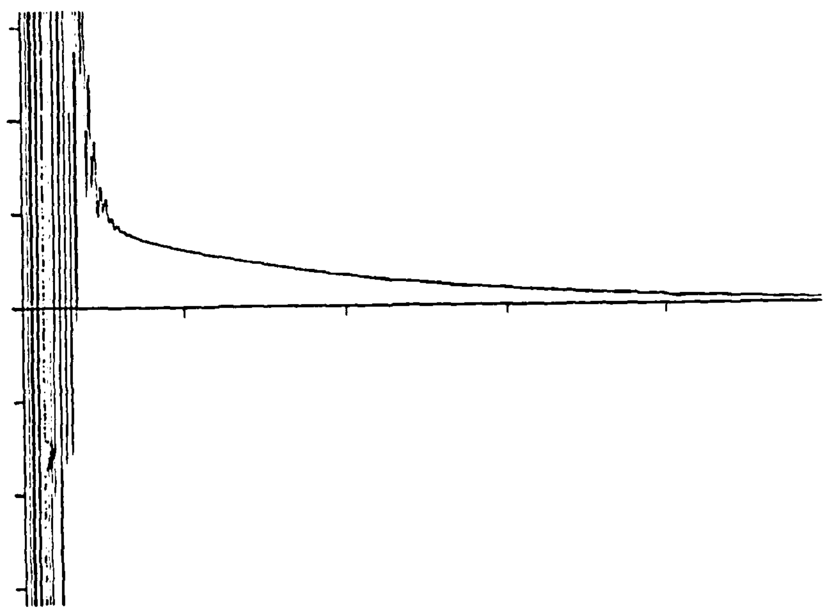


Fig. 33. Example 2 input force/time K=9

Example 3

In this example the control system will be applied to basically the same simply supported beam, but the length will be increased to $L=10$ units. All the other physical qualities of the beam will remain the same as in the earlier examples. The structure will consist of $N_s=10$ modes and the model will have $N=6$ modes. The damping ratio is $\alpha=-4.0$ and the stiffness coefficient $\beta=0$. The observed point is selected as $x=2.1$ units. The time tics on the plots are at 4 second intervals and the displacement and force tics are at 0.2 unit intervals. When $K=8$ the input points are at $x=0.9$, 2.1, 3.1, 4.2, 5.3, 6.1, 7.3, and 8.2 units. When $K=10$ the points $x=9.3$, 9.7 are added and for $K=14$ the points $x=4.75$, 1.8, 0.5, and 3.55 units are included. When $K=17$ the final input points are $x=6.45$, 2.7 and 0.7 units. The displacement responses for the model and the structure are shown in Figures 34-41. By examining the responses we note that by $K=8$ the model appears to have already reached the ideal case. If we examining Table 3 we can see that the closed-loop eigenvalues are not close enough to the ideal eigenvalues and the structure still has a very large residual effect. However, by $K=17$ the structure response closely resembles the ideal model with only a small amount of residual remaining.

Table 3. Example 3 closed-loop eigenvalues

IDEAL	K=8	K=17
-.0024367	-.0024368	-.0024367
-.039351	-.039388	-.039356
-.20808	-.20968	-.20827
-.77267	-.81122	-.776622
-3.2273	-3.0980	-3.2110
-3.7919	-3.7699	-3.7885
-3.9606	-3.9577	-3.9601
-3.9976	-3.9974	-3.9975
-2.00±i1.445	-1.85±i1.659	-1.98±i1.464
-2.00±i2.937	-1.61±i3.119	-1.97±i2.956

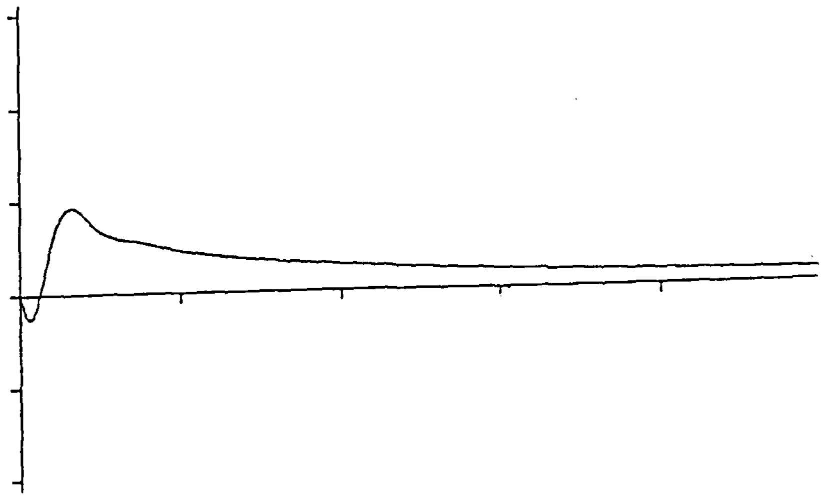


Fig. 34. Example 3 model displacement/time K=8

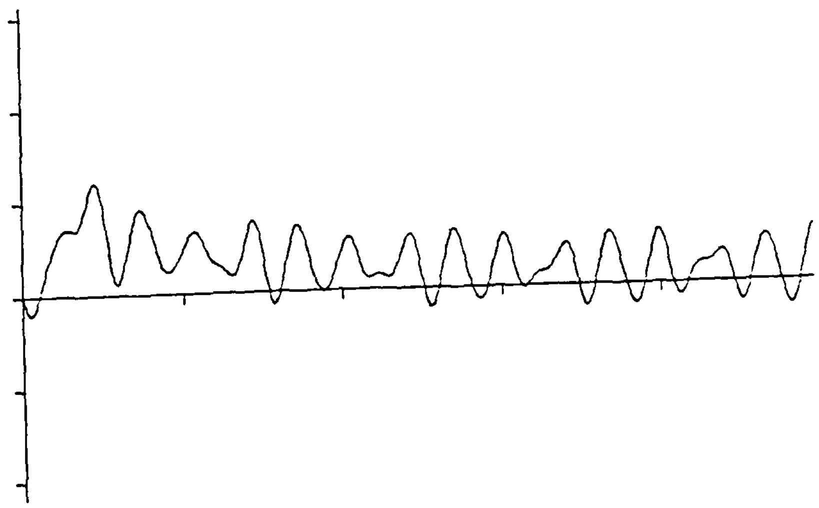


Fig. 35. Example 3 structure displacement/time K=8

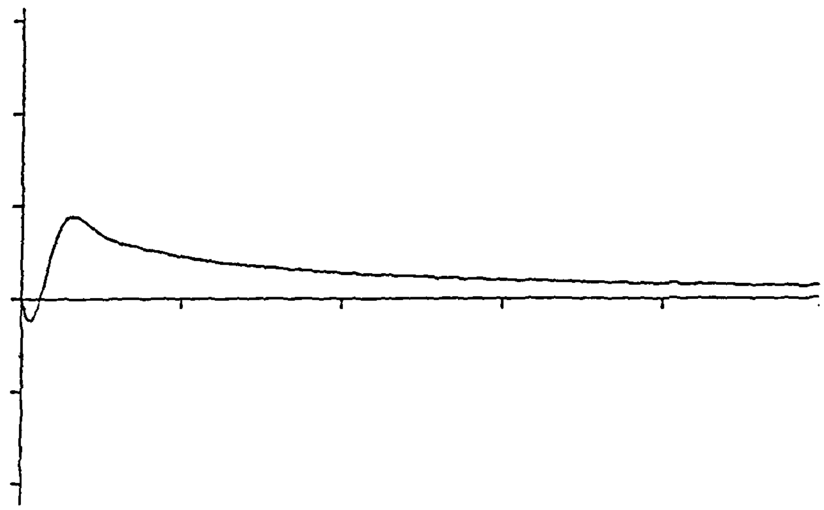


Fig. 36. Example 3 model displacement/time K=10

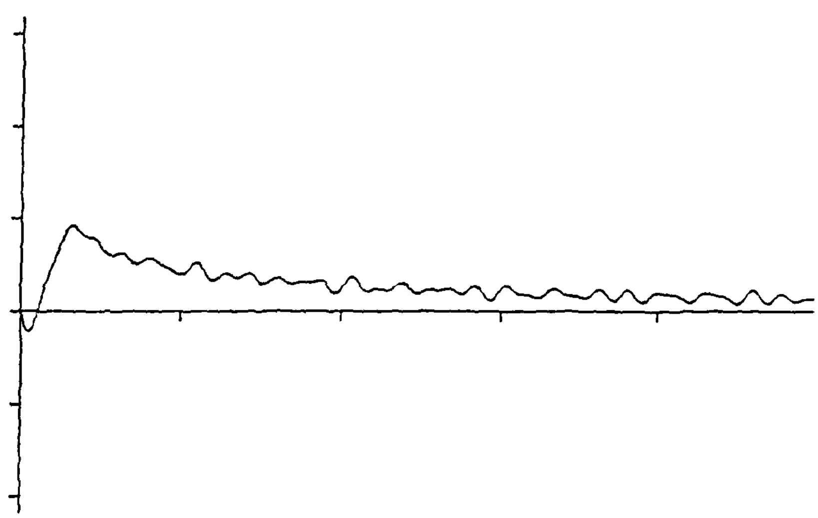


Fig. 37. Example 3 structure displacement/time K=10

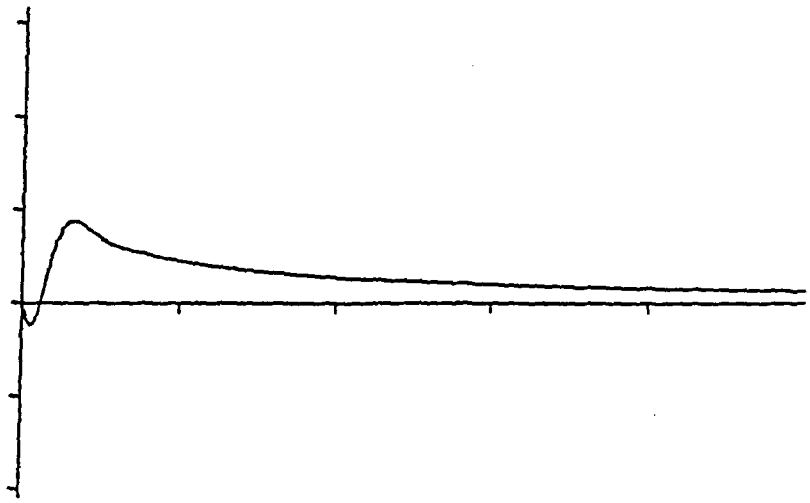


Fig. 38. Example 3 model displacement/time K=14

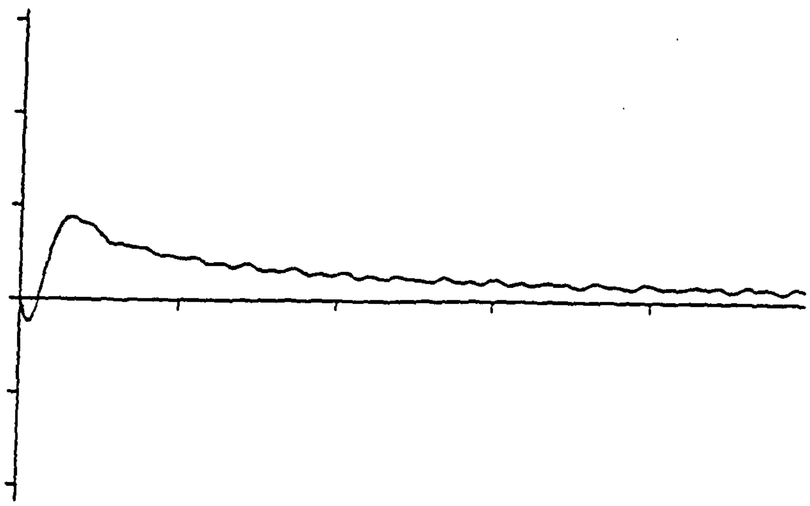


Fig. 39. Example 3 structure displacement/time K=14

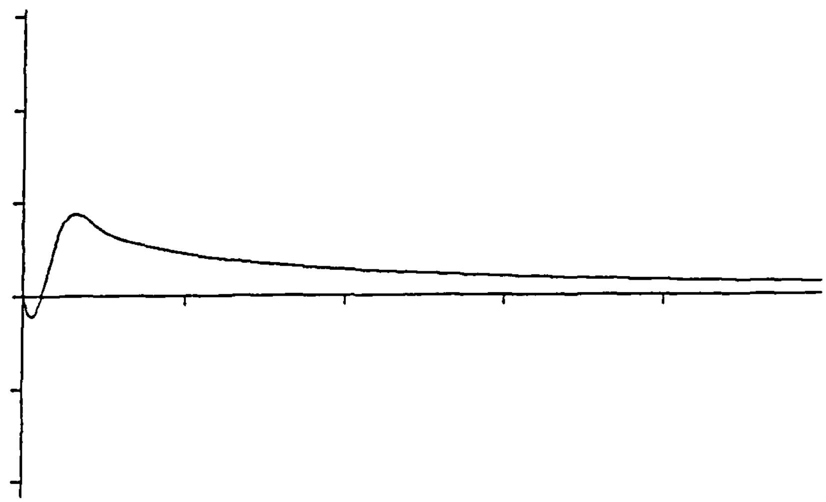


Fig. 40. Example 3 model displacement/time K=17

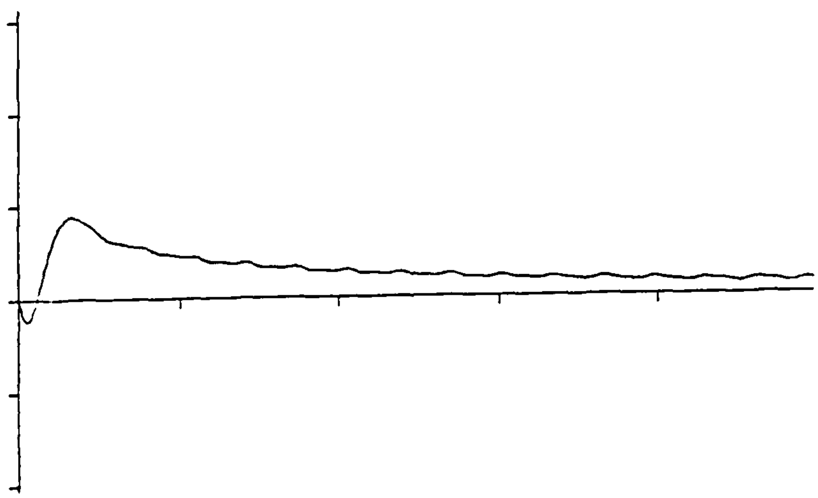


Fig. 41. Example 3 structure displacement/time K=17

CHAPTER III

CONTROL OF AN UNDAMPED TRUSS

Introduction

In this chapter the closed-loop feedback control method and the open-loop method will be applied to a finite degrees of freedom undamped truss structure. We will examine the structure and the implementation of the control system on this structure, then we will look at two closed-loop and two open-loop examples.

Implementation

The truss that we will be applying the control system to [12] is a two-dimensional undamped structure (Figure 42) composed of seven beams with unit mass distribution, unit stiffness EA=1, and unit length L=1. The structures left side is pinned to a wall at both joints. The lower left joint where beams 1 and 2 are connected is the origin for the global coordinates x and y. The generalized coordinates used to describe the displacement at each joint are U_1, U_2, ... U_{10}, but since two of the joints are pinned their displacements will be zero and their generalized coordinates are dropped from the final equation.

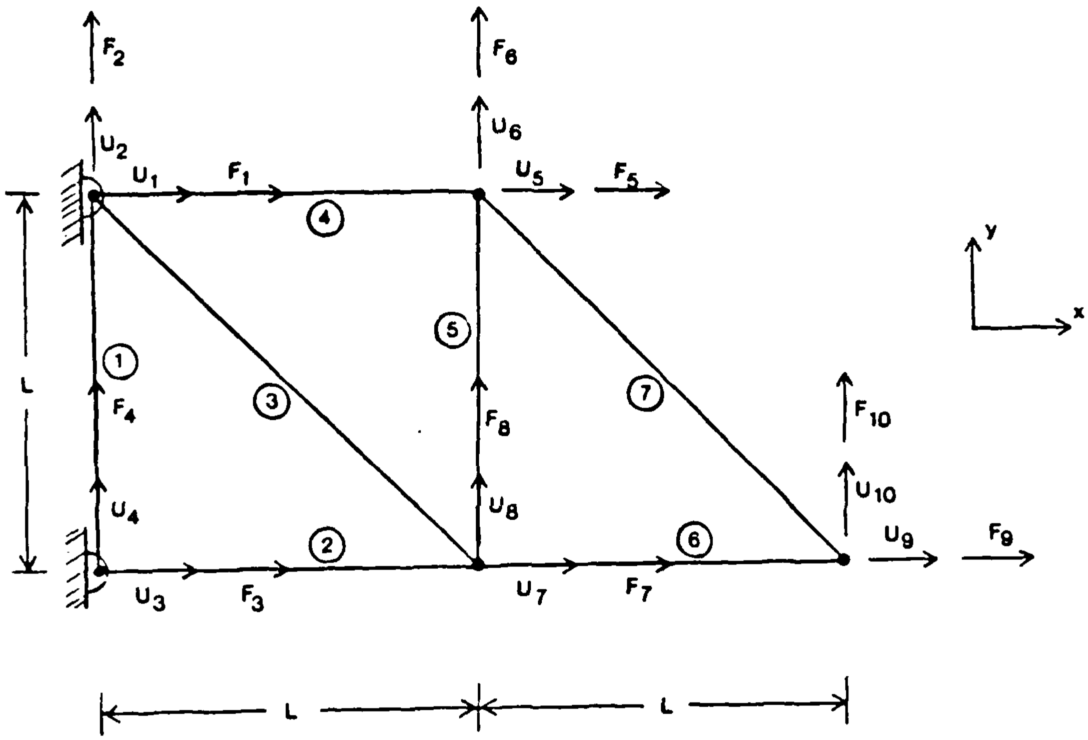


Fig. 42. An undamped truss

The equation of motion of the truss has the form

$$M\ddot{U} + KU = DF \tag{76}$$

where M is the mass matrix, K is the stiffness matrix, F is the input force vector, and D is a force distribution matrix. The mass and stiffness matrix have the form

$$M = mL/6\sqrt{2} \begin{vmatrix} 2+2\sqrt{2} & -2 & 0 & 0 & 1 & -1 \\ -2 & 2+2\sqrt{2} & 0 & \sqrt{2} & -1 & 1 \\ 0 & 0 & 2+4\sqrt{2} & -2 & \sqrt{2} & 0 \\ 0 & \sqrt{2} & -2 & 2+2\sqrt{2} & 0 & 0 \\ 1 & -1 & \sqrt{2} & 0 & 2+2\sqrt{2} & -2 \\ -1 & 1 & 0 & 0 & -2 & 2 \end{vmatrix}, \tag{77}$$

$$K = EA/2\sqrt{2}L \begin{vmatrix} 1+2\sqrt{2} & -1 & 0 & 0 & -1 & 1 \\ 0 & 1+2\sqrt{2} & 0 & -2\sqrt{2} & 1 & -1 \\ 0 & 0 & 1+4\sqrt{2} & -1 & -2\sqrt{2} & 0 \\ 0 & -2\sqrt{2} & -1 & 1+2\sqrt{2} & 0 & 0 \\ -1 & 1 & -2\sqrt{2} & 0 & 1+2\sqrt{2} & -1 \\ 1 & -1 & 0 & 0 & -1 & 1 \end{vmatrix}. \tag{78}$$

The displacement and the force vector have the form

$$U = \{U_5, U_6, \ldots U_{10}\}^T \tag{79}$$

$$F = \{F_5, F_6, \ldots F_{10}\}^T. \tag{80}$$

The closed-loop control on the truss is implemented in the same manner as on the beam with the exception that the input forces for these example are only being applied at the joints and in the direction of the structure's degrees of freedom as depicted in Figure 42. The computer simulation of the closed-loop system is accomplished through the program PNTCNTF (Appendix B).

The open-loop control is accomplished by controlling the model without any feedback from the structure on the effect of the residual modes. When the open-loop control method is implemented the model response for any number of

input points will always be the ideal case, but the structure response will move towards the ideal model as the number of inputs increase. The computer simulation of the open-loop system is accomplished by modifying the program PNTCNTF so that there is no feedback from the structure to the model.

For all the examples the initial modal state disturbances are chosen as $q_r(0)=0$ and $\dot{q}_r(0)=1.0$ units/sec, $r=1,2,...N$. The joint chosen to observe the model and structure responses will be the upper right joint where the beams 4 , 5 , and 7 are connected. Since the structure is two dimensional we will examine the displacement and input force in both the x and the y direction. The horizontal axis on all the plots will reflect the time and the tics at 2 second intervals. The vertical axis will reflect either the displacement or the input force with the tics at 0.25 unit intervals. The structure has a finite number of modes, $N_s=6$, and the reduced order model controls $N=n$ modes where $N<N_s$. The number of inputs will be referred to as K.

<u>Example 4</u>

In this closed-loop control system example, the reduced order model will consist of the first two modes of the structure $N=2$. The damping coefficient is selected as $\alpha=-2$ and the stiffness coefficient is selected as $\beta=0$.

When K=2 the input forces F_5 and F_6 are being applied. When K=4 the input forces F_7 and F_8 are added and for K=6 the input forces F_9 and F_{10} are included. The model and structure displacement responses are shown in Figures 43-54. As in the previous examples as the number of inputs increase both the actual model and the structure approach the ideal model response which is reached when K=6. If we look at the ideal gain matrix for control in the x direction we see that it has the form

$$G = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & -2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -2 \end{bmatrix}. \tag{81}$$

When K=2 the non-ideal gain matrix is

$$G_n = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & -0.07 & 0 & -0.05 \\ 0 & 0 & 0 & 0 \\ 0 & -0.05 & 0 & -0.04 \end{bmatrix}. \tag{82}$$

When K=4 the non-ideal gain matrix is

$$G_n = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & -1.20 & 0 & -0.59 \\ 0 & 0 & 0 & 0 \\ 0 & -0.59 & 0 & -0.69 \end{bmatrix}. \tag{83}$$

When K=6 the non-ideal gain matrix is

$$G_n = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & -2.0 & 0 & 6.1\times10^{-14} \\ 0 & 0 & 0 & 0 \\ 0 & 6.1\times10^{-14} & 0 & -2.0 \end{bmatrix}. \tag{84}$$

By examining the gain matrices it can be easily seen that as the number of inputs increase the actual model, that includes feedback from the structures residual modes, approaches the ideal model. By examining Table 4 we can

see the same thing happens with the closed-loop eigenvalues. In Figures 55 and 56 we can also see that the F_6 input force approaches the ideal case as the number of inputs increase.

Table 4. Example 4 closed-loop eigenvalues

IDEAL	K=2	K=4	K=6
-1.0±i2.267	-0.04±i2.985	-0.24±i2.609	-1.0±i2.267
-1.0±i2.814	-0.20±i2.479	-0.70±i2.736	-1.0±i2.814

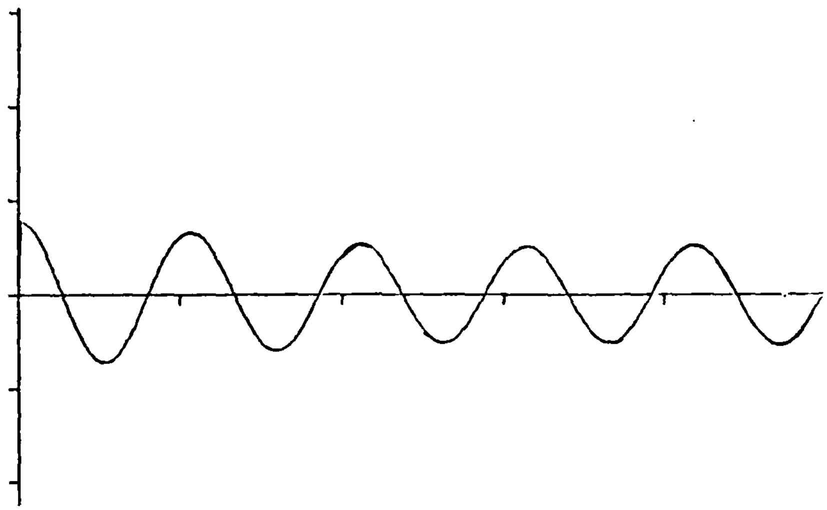


Fig. 43. Example 4 model x displacement/time K=2

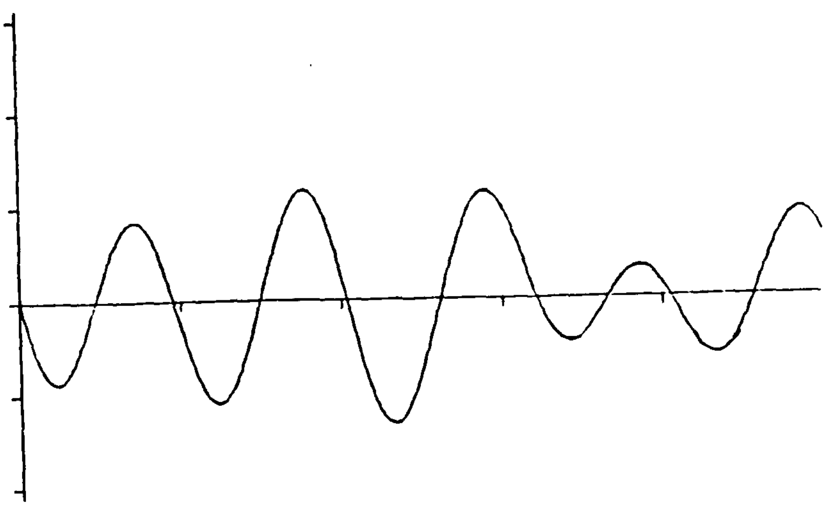


Fig. 44. Example 4 structure x displacement/time K=2

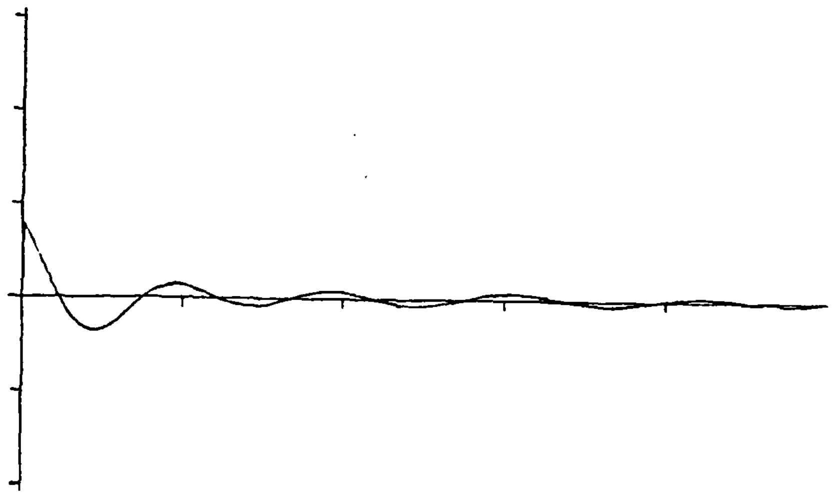


Fig. 45. Example 4 model x displacement/time K=4

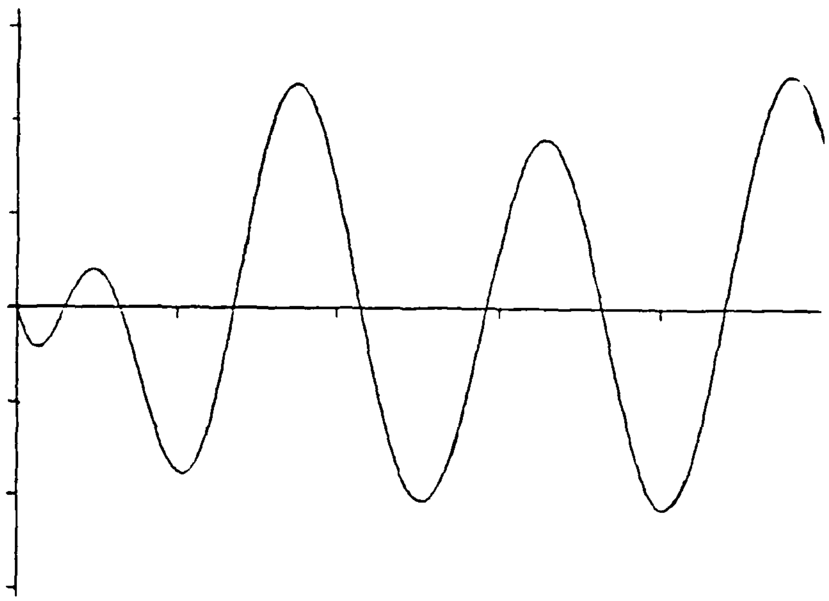


Fig. 46. Example 4 structure x displacement/time K=4

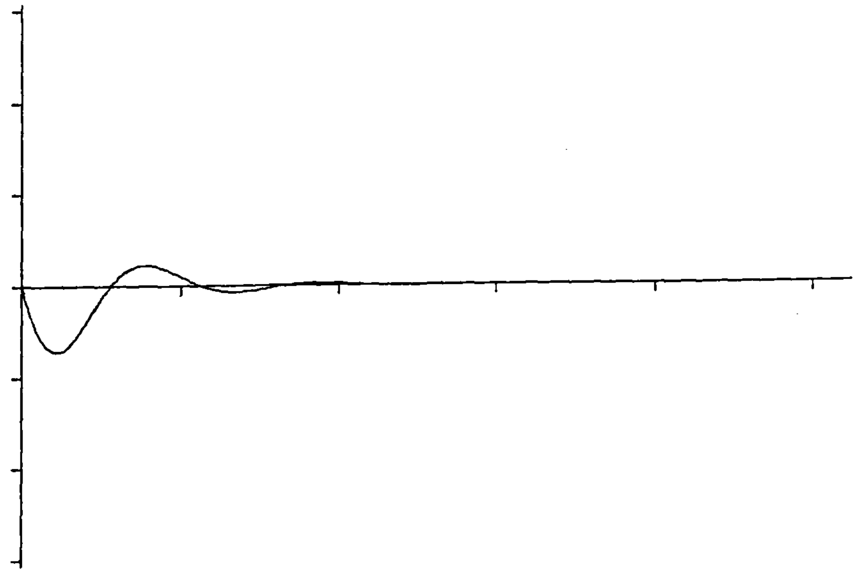


Fig. 47. Example 4 model x displacement/time K=6

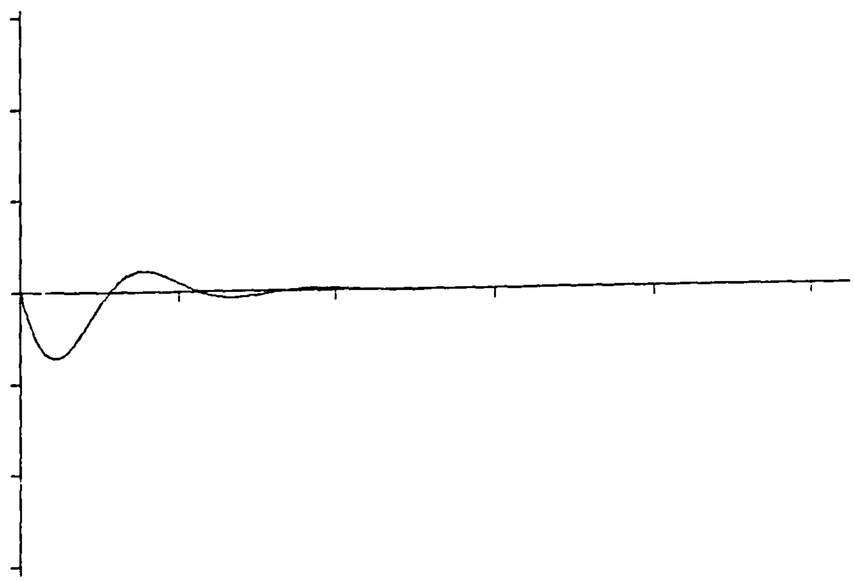


Fig. 48. Example 4 structure x displacement/time K=6

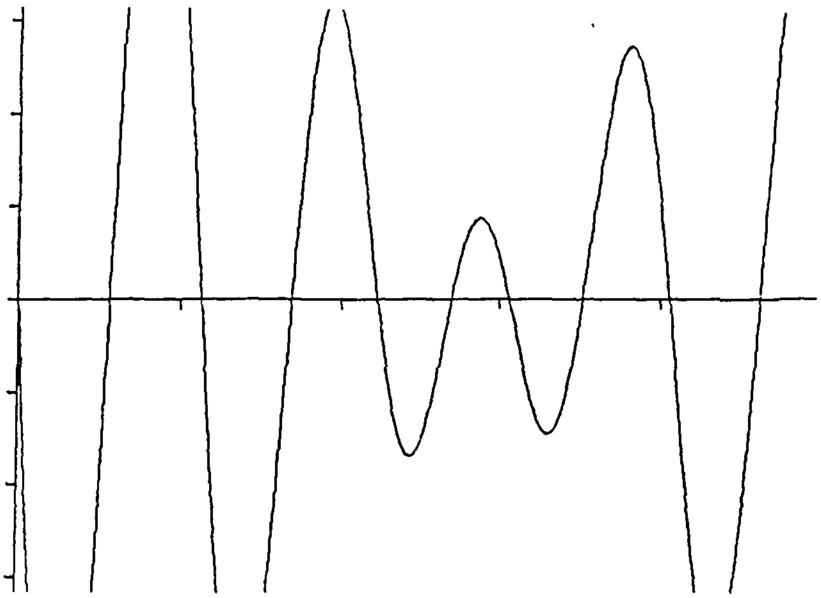


Fig. 49. Example 4 model y displacement/time K=2

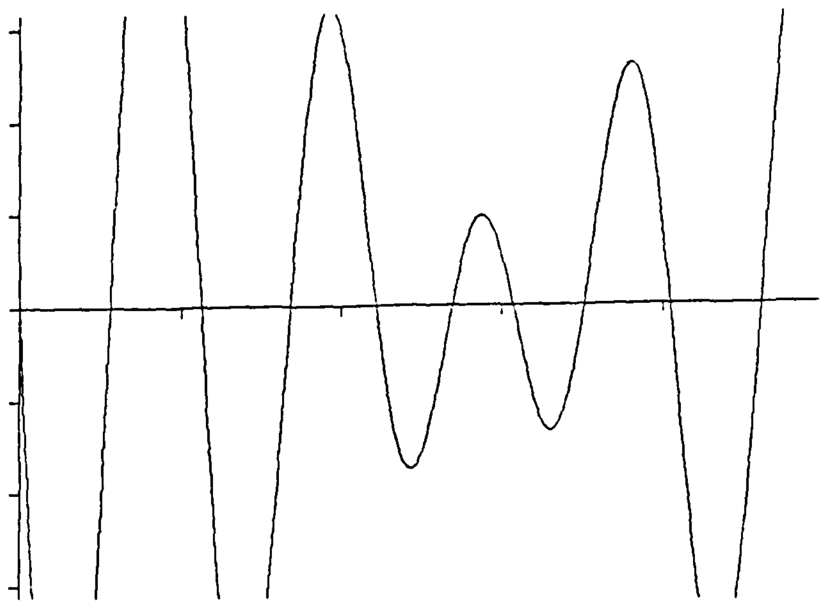


Fig. 50. Example 4 structure y displacement/time K=2

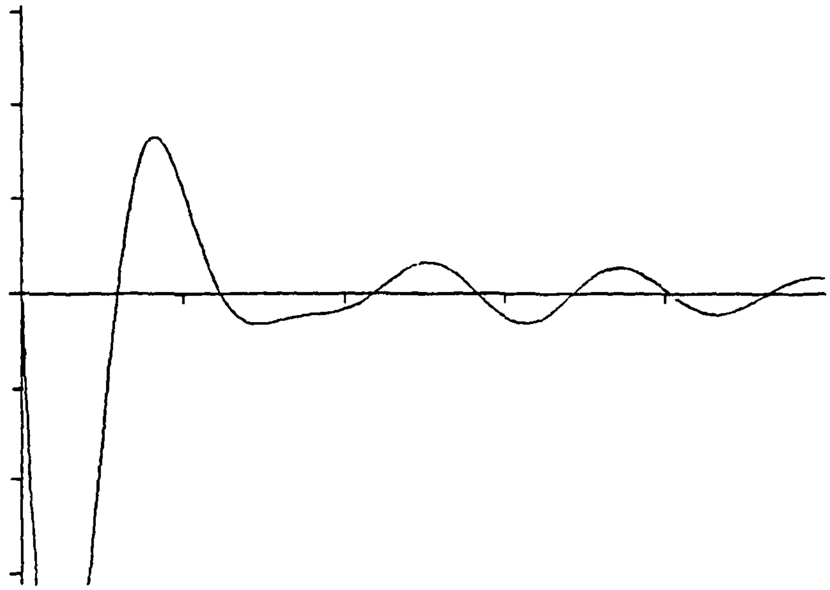


Fig. 51. Example 4 model y displacement/time K=4

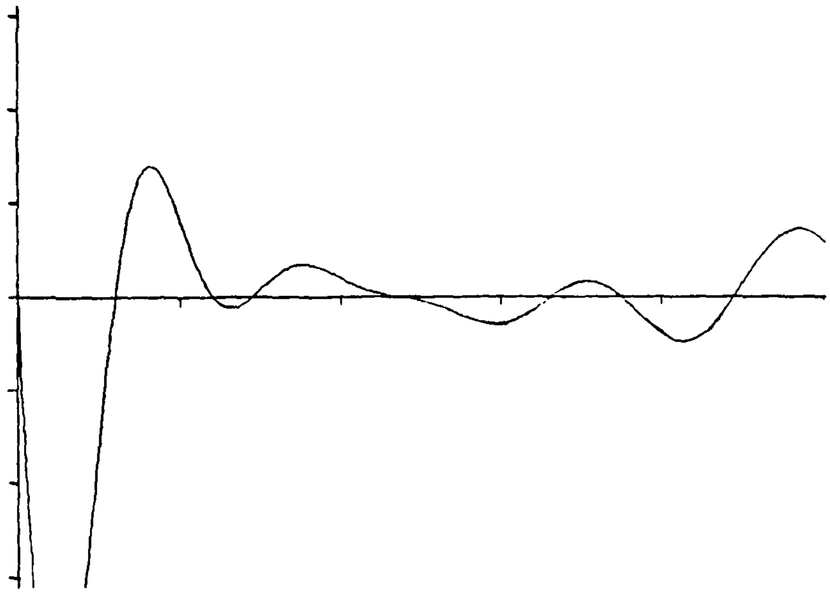


Fig. 52. Example 4 structure y displacement/time K=4

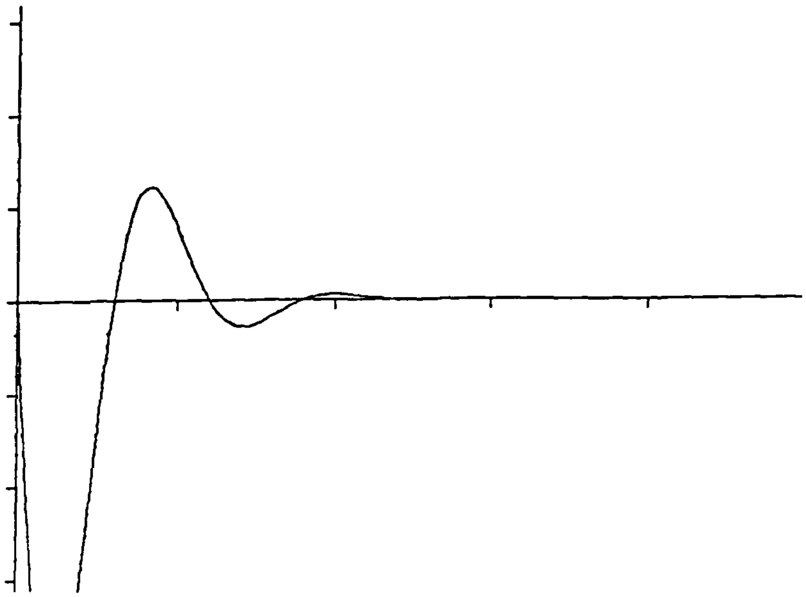


Fig. 53. Example 4 model y displacement/time K=6

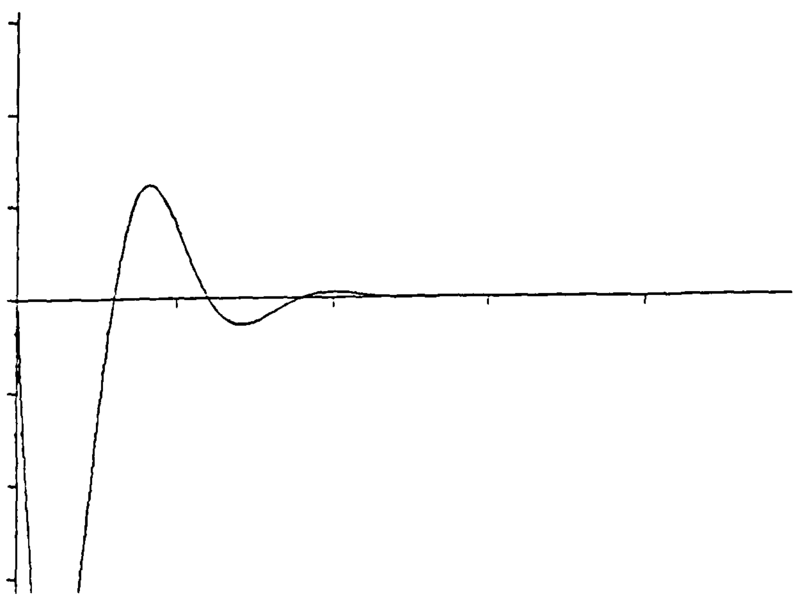


Fig. 54. Example 4 structure y displacement/time K=6

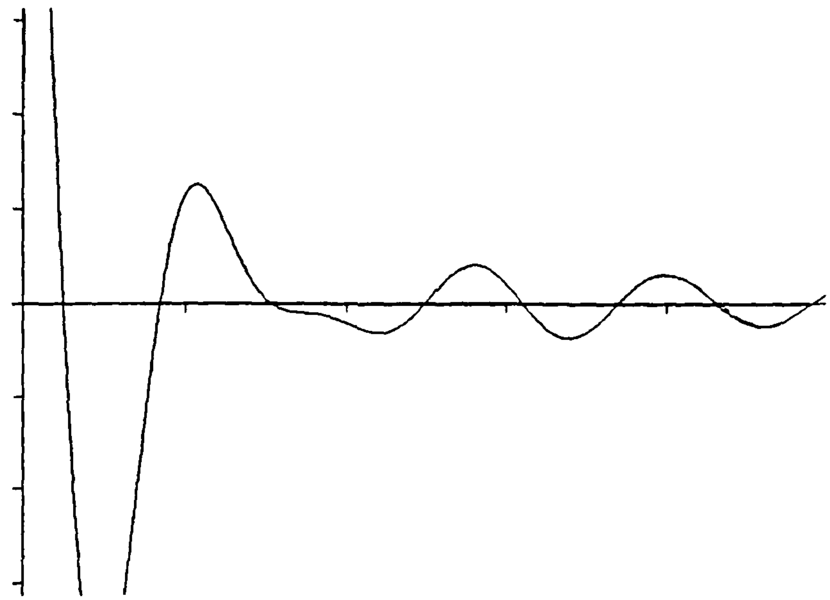


Fig. 55. Example 4 F_6 input force/time K=4

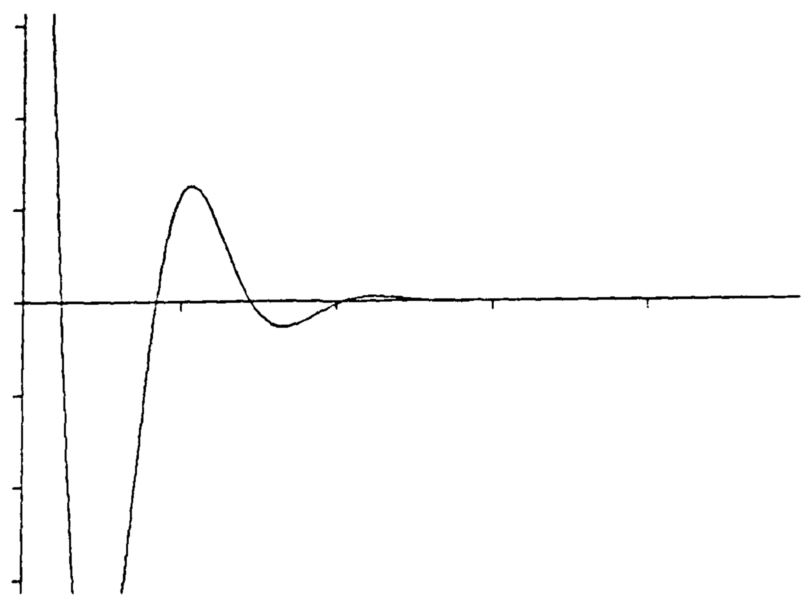


Fig. 56. Example 4 F_6 input force/time K=6

Example 5

This closed-loop example is very similar to Example 4 except that the reduced order model is increased to include the first three modes of the structure N=3 and the damping coefficient is increased to $\alpha=-5$. All the other parameters remain the same as in Example 4 including the input points. The displacement response for the model and the structure in both the x and y direction can be seen in Figures 57-68.

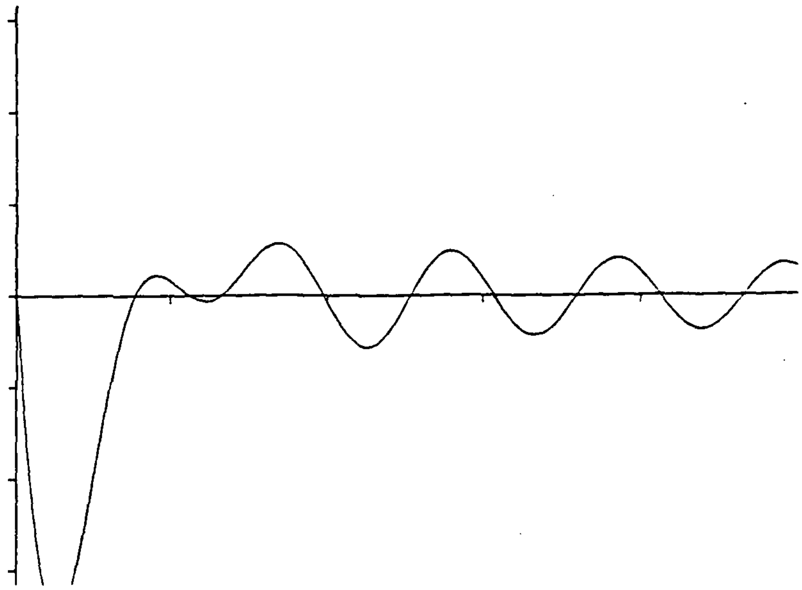


Fig. 57. Example 5 model x displacement/time K=2

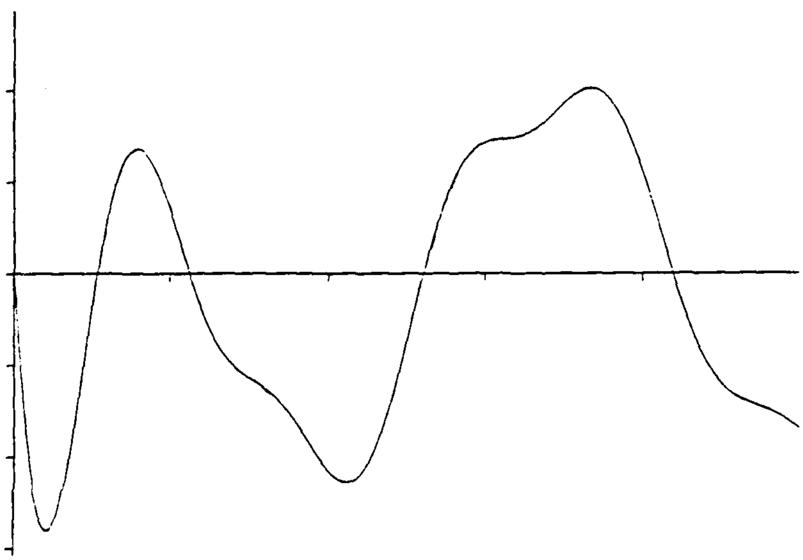


Fig. 58. Example 5 structure x displacement/time K=2

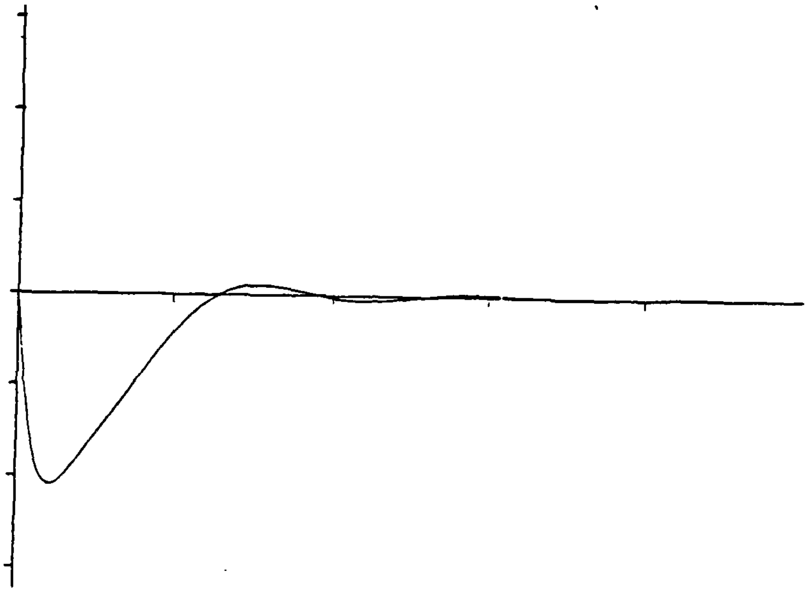


Fig. 59. Example 5 model x displacement/time K=4

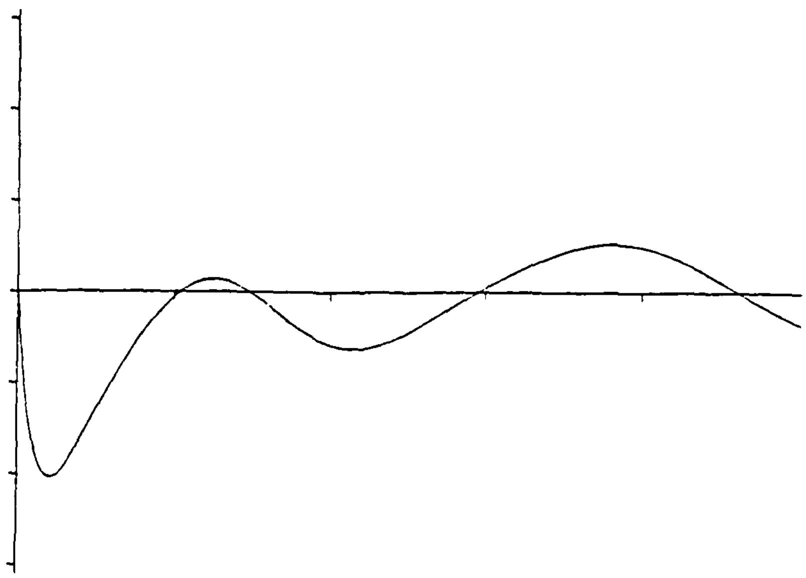


Fig. 60. Example 5 structure x displacement/time K=4

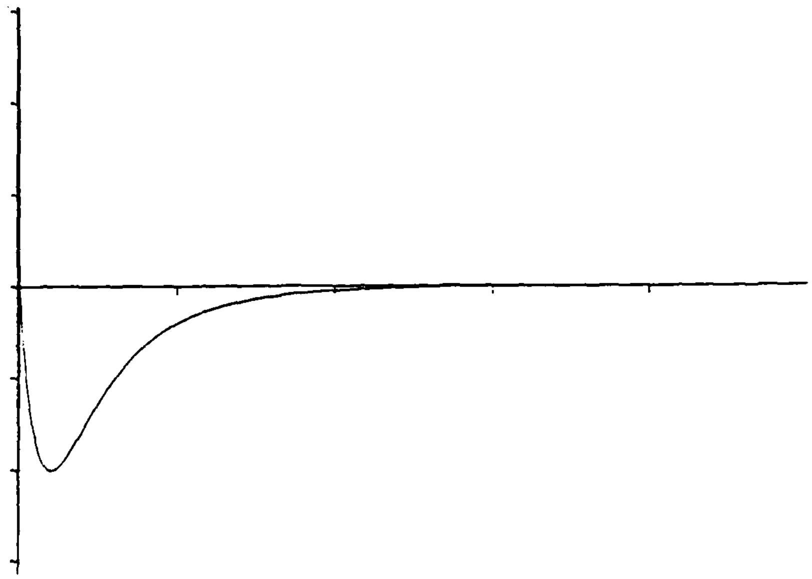


Fig. 61. Example 5 model x displacement/time K=6

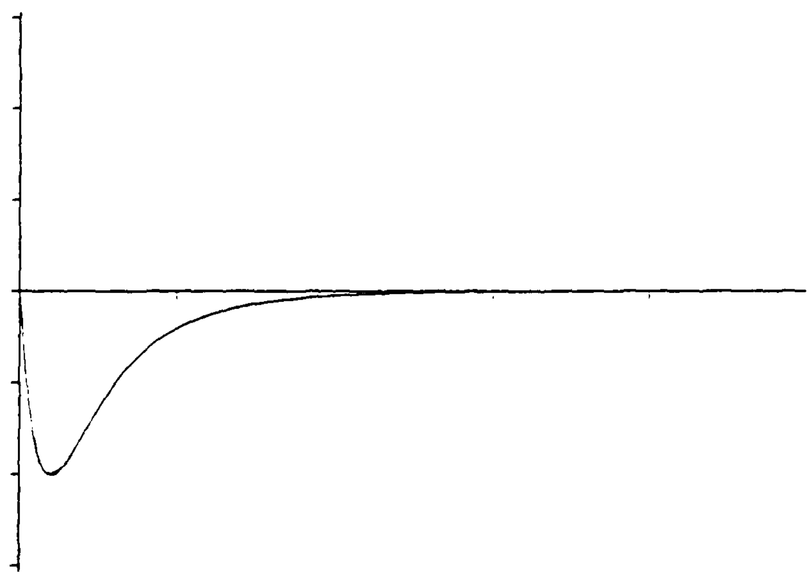


Fig. 62. Example 5 structure x displacement/time K=6

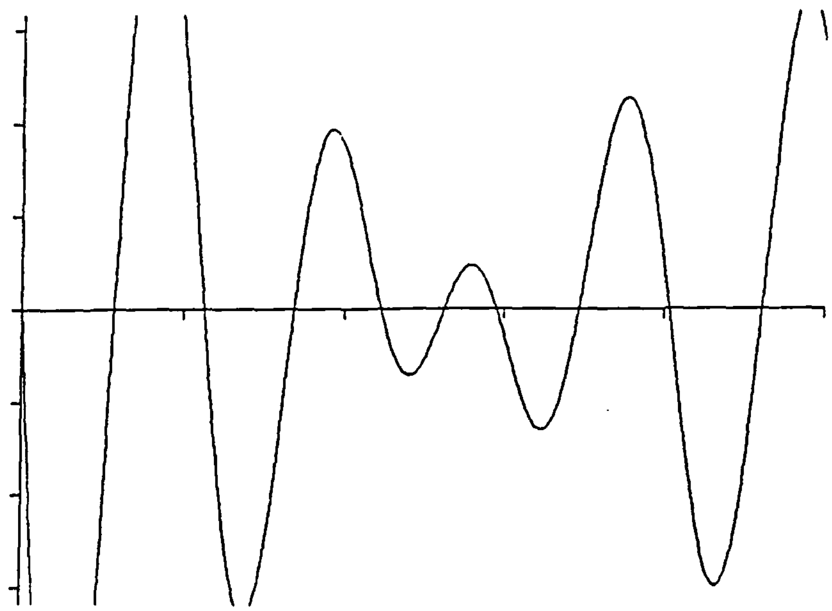


Fig. 63. Example 5 model y displacement/time K=2

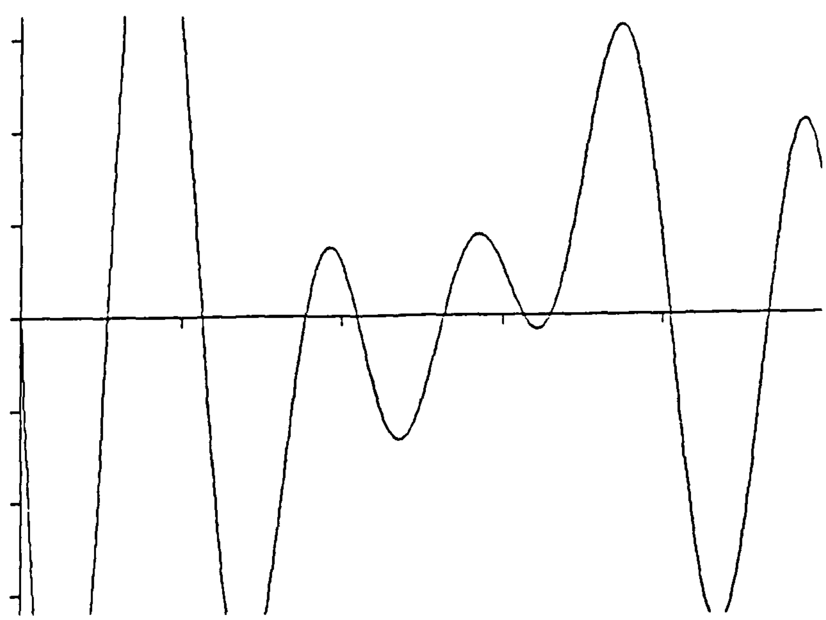


Fig. 64. Example 5 structure y displacement/time K=2

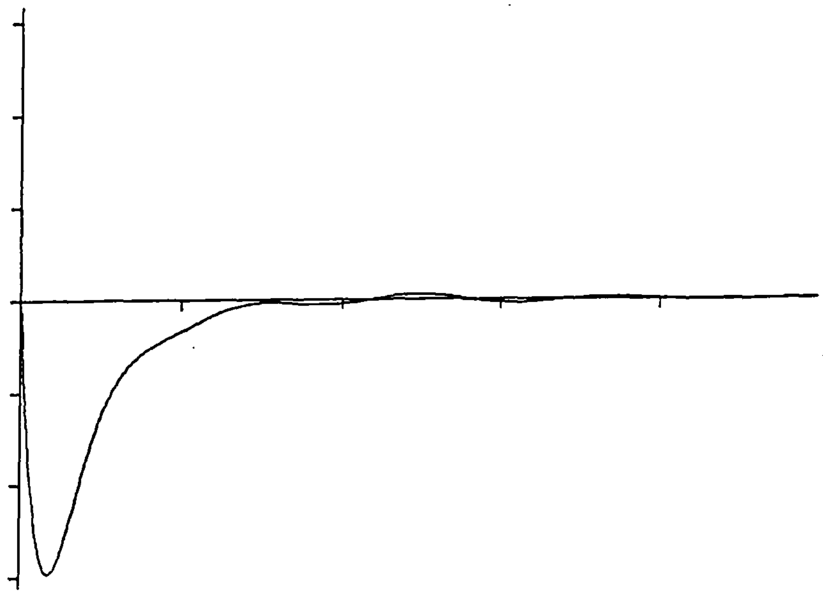


Fig. 65. Example 5 model y displacement/time K=4

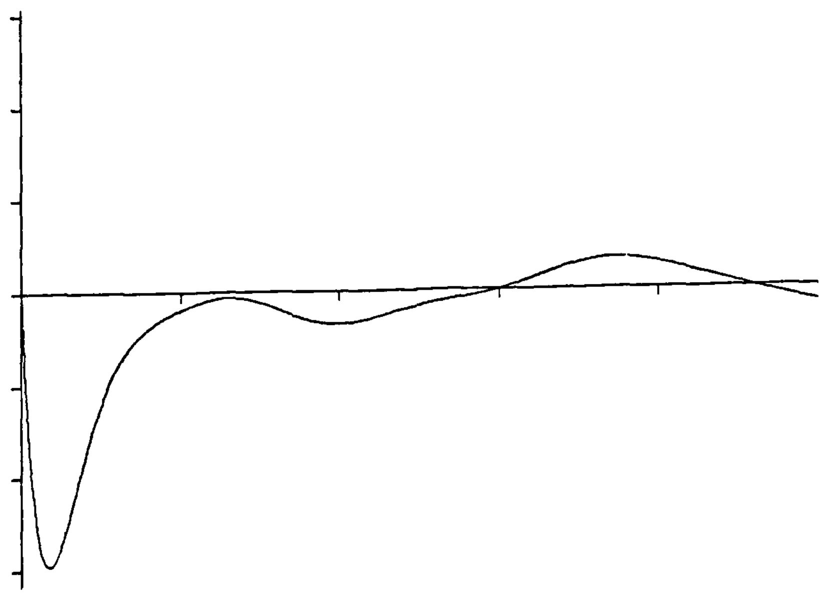


Fig. 66. Example 5 structure y displacement/time K=4

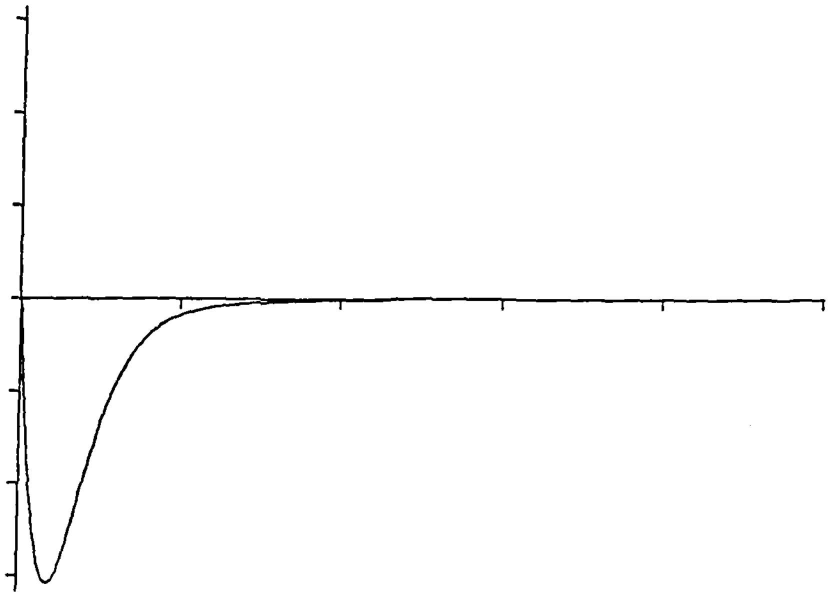


Fig. 67. Example 5 model y displacement/time K=6

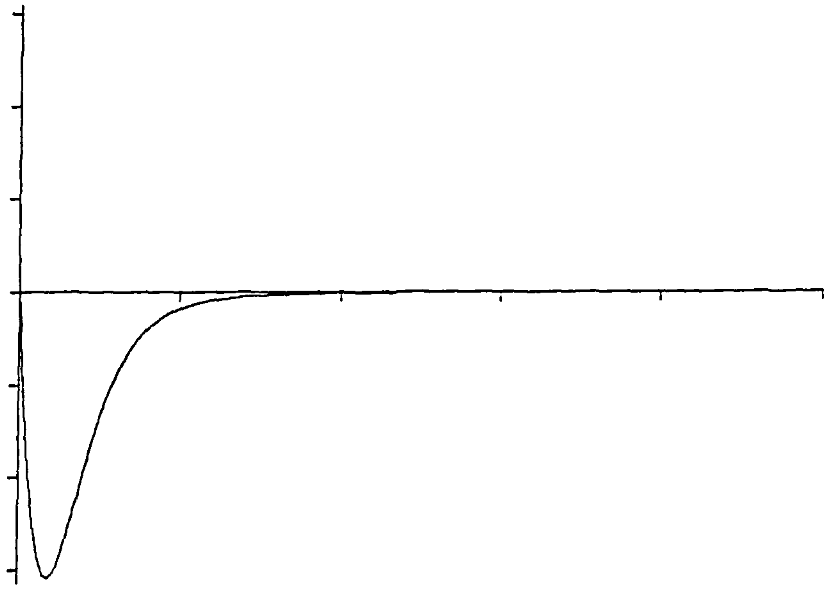


Fig. 68. Example 5 structure y displacement/time K=6

<u>Example 6</u>

In this example we will apply the open-loop control system to the undamped truss. To get a good comparison of the open-loop method and closed-loop method we will apply the open-loop control to the same problem as in Example 4. Therefore, the reduced order model will be N=2 and the damping and stiffness coefficients will be $\alpha=-2$ and $\beta=0$. The inputs will be the same as in Example 4. When open-loop control is used the effect of the residual modes on the structure is not feed back to the reference controlled model, equation (32). Therefore, the reference model response will always reflect the ideal case, this can be seen in Figures 69 and 73. The structure displacement responses in both the x and y direction can be seen in Figures 70-72 and 74-76. In examining these figures we can see that the effect of the open-loop control on the structure is not as strong as the closedloop control when the input number is small. However, when enough input points are used the structure will resemble the ideal model and the effect of the residual modes is eliminated. In Figures 77 and 78 we can see that the input force response decreases as the number of inputs increase.

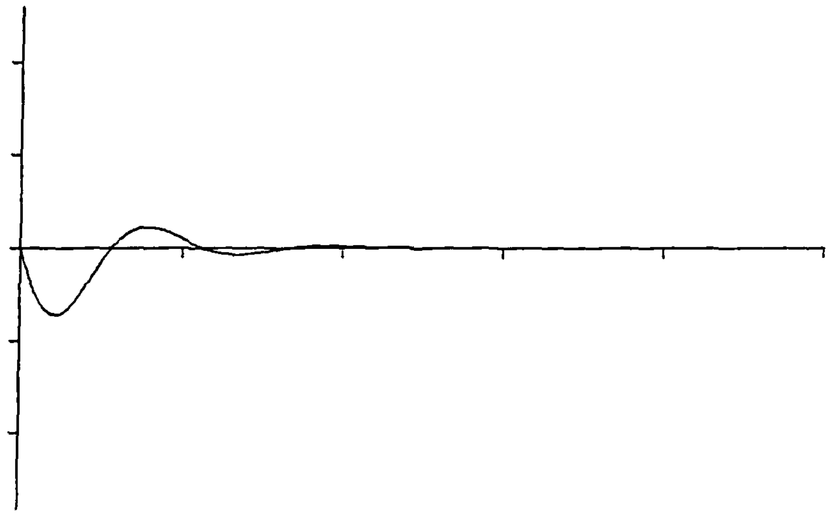


Fig. 69. Example 6 ideal model x displacement/time

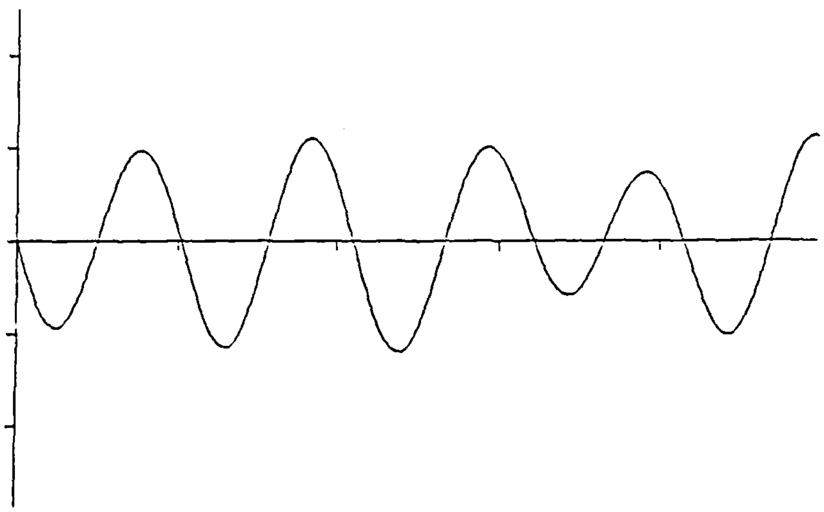


Fig. 70. Example 6 structure x displacement/time K=2

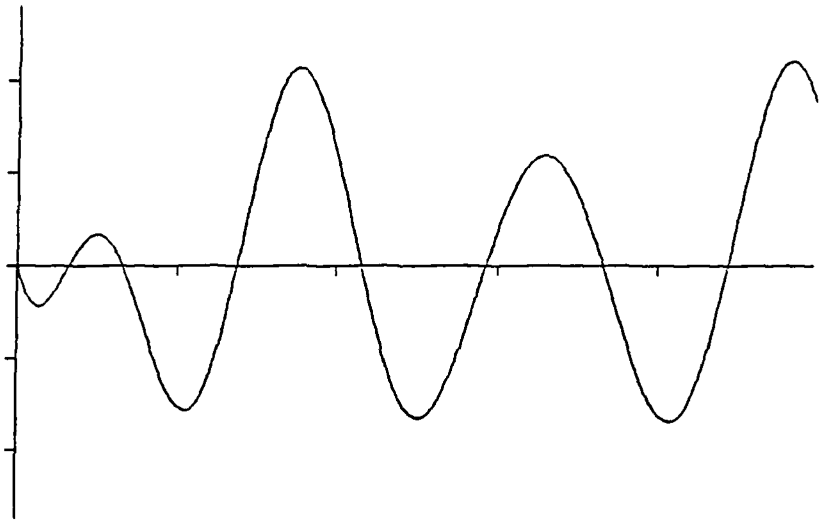


Fig. 71. Example 6 structure x displacement/time K=4

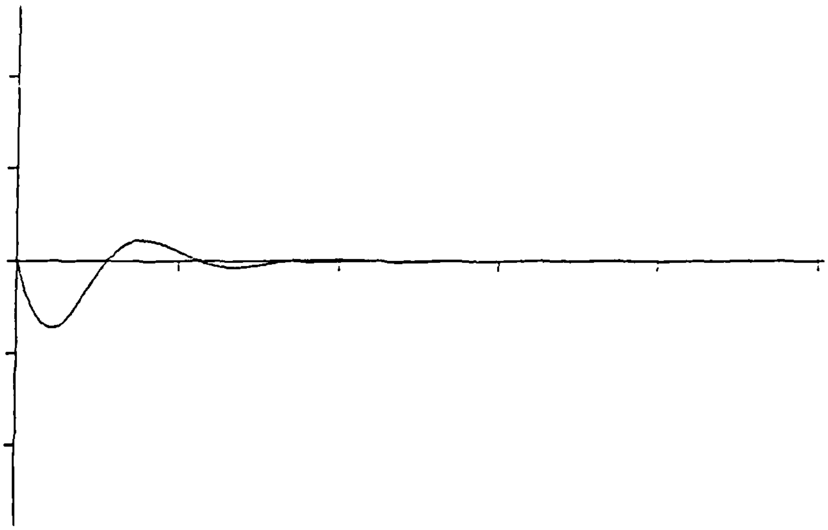


Fig. 72. Example 6 structure x displacement/time K=6

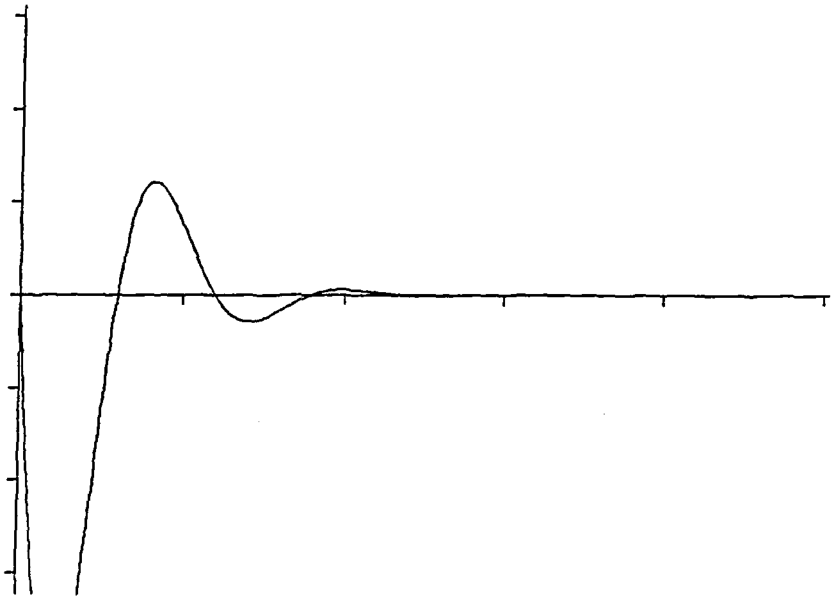


Fig. 73. Example 6 ideal model y displacement/time

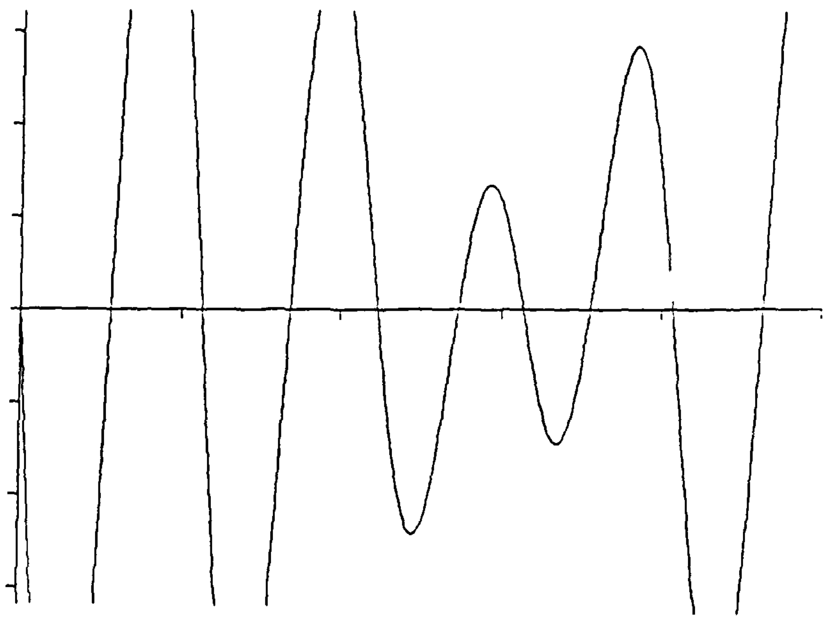


Fig. 74. Example 6 structure y displacement/time K=2

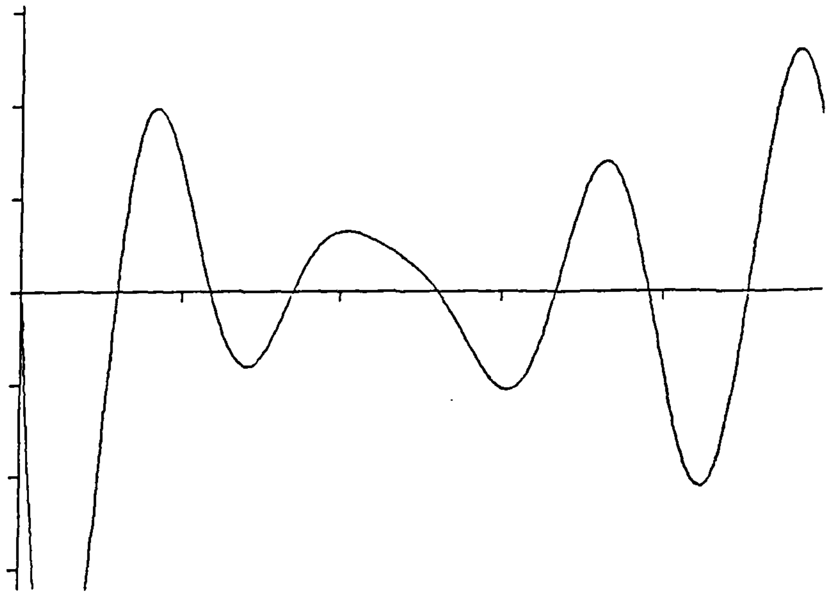


Fig. 75. Example 6 structure y displacement/time K=4

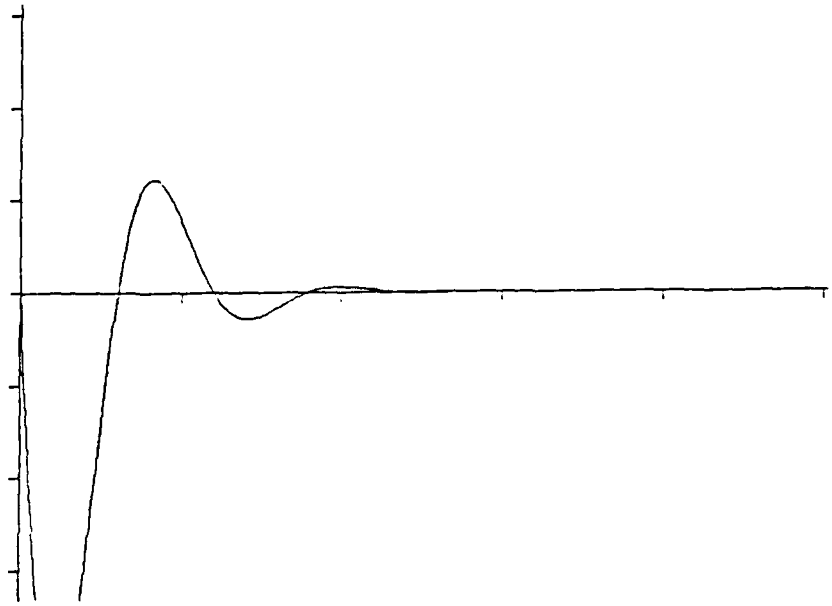


Fig. 76. Example 6 structure y displacement/time K=6

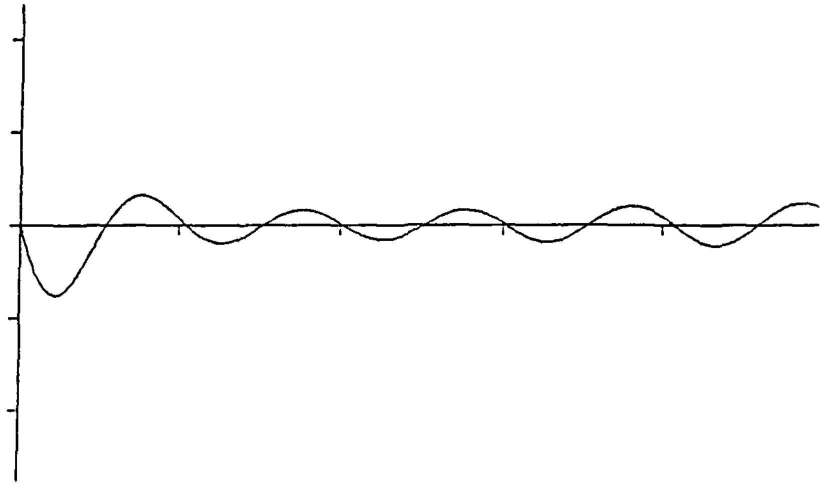


Fig. 77. Example 6 F_5 input force /time K=4

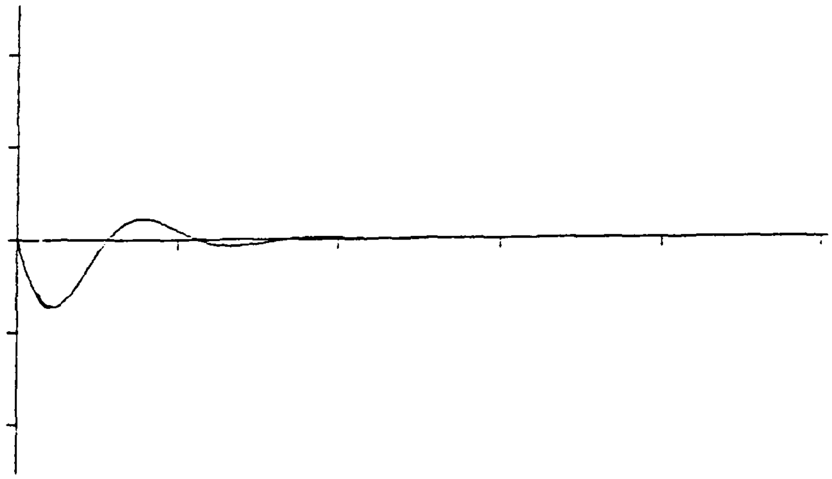


Fig. 78. Example 6 F_5 input force/time K=6

Example 7

In this example we will apply the open-loop control system to the same problem as in Example 5. The reduced order model will consist of the first three modes of the structure N=3 and the damping and stiffness coefficients will be $\alpha=-5$ and $\beta=0$. All the input points remain the same as in Example 5. The ideal model displacement response in both the x and y direction can be seen in Figures 79 and 83. The structure displacement responses in both the x and y direction can be seen in Figures 80-82 and 84-86.

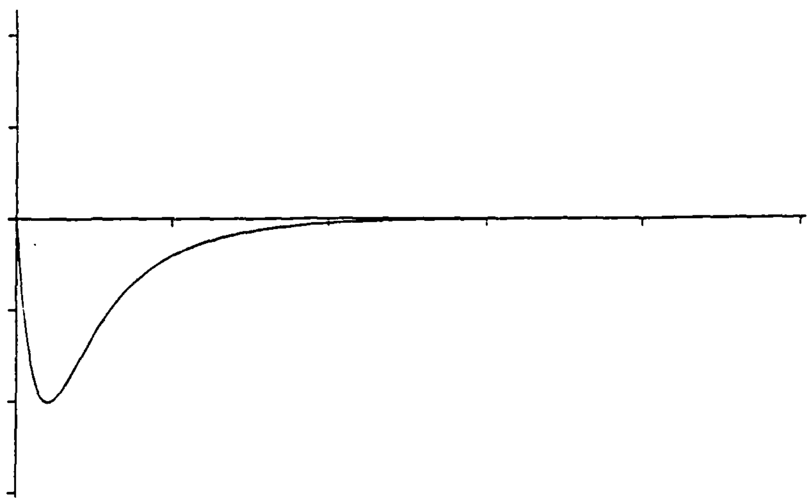


Fig. 79. Example 7 ideal model x displacement/time

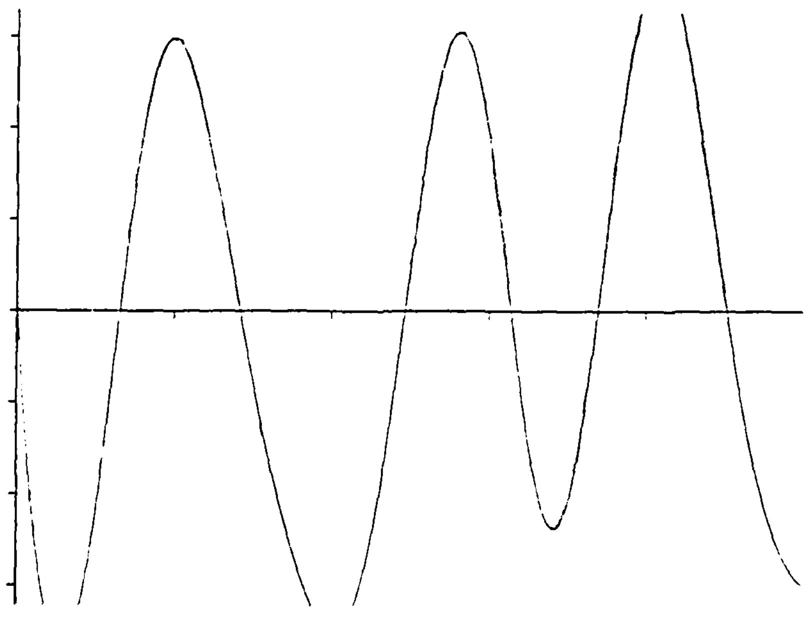


Fig. 80. Example 7 structure x displacement/time K=2

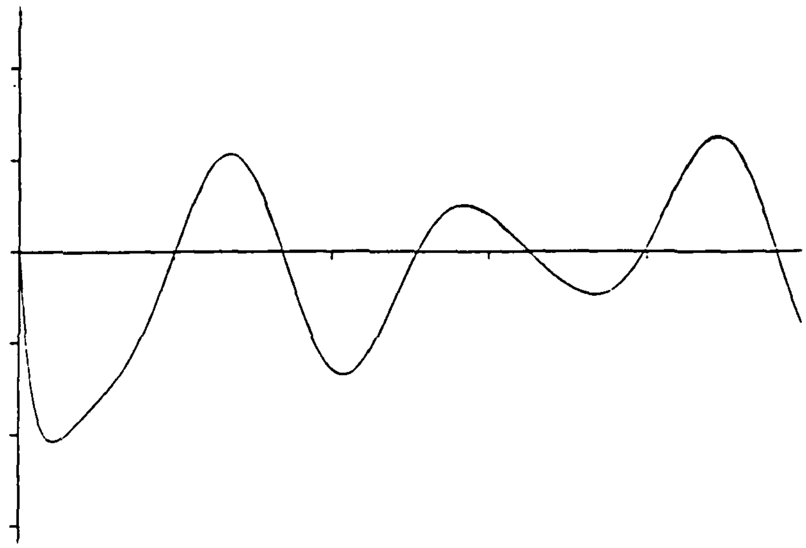


Fig. 81. Example 7 structure x displacement/time K=4

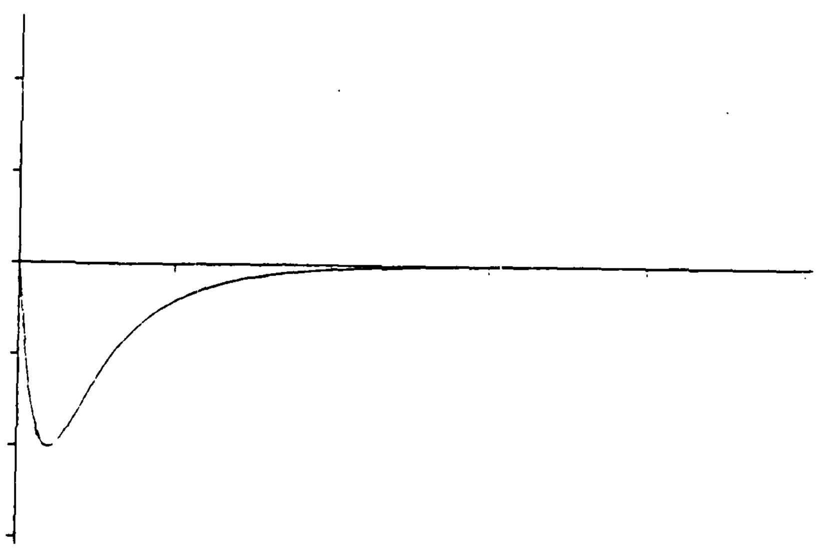


Fig. 82. Example 7 structure x displacement/time K=6

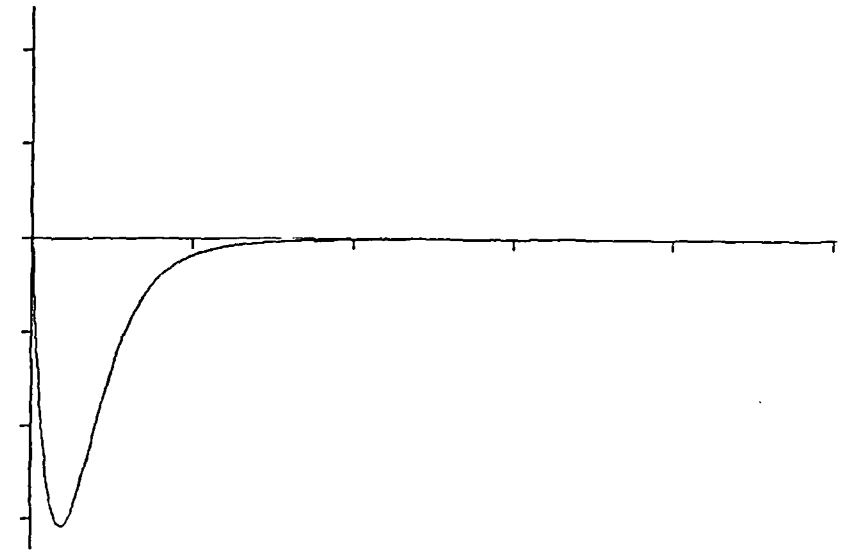


Fig. 83. Example 7 ideal model y displacement/time

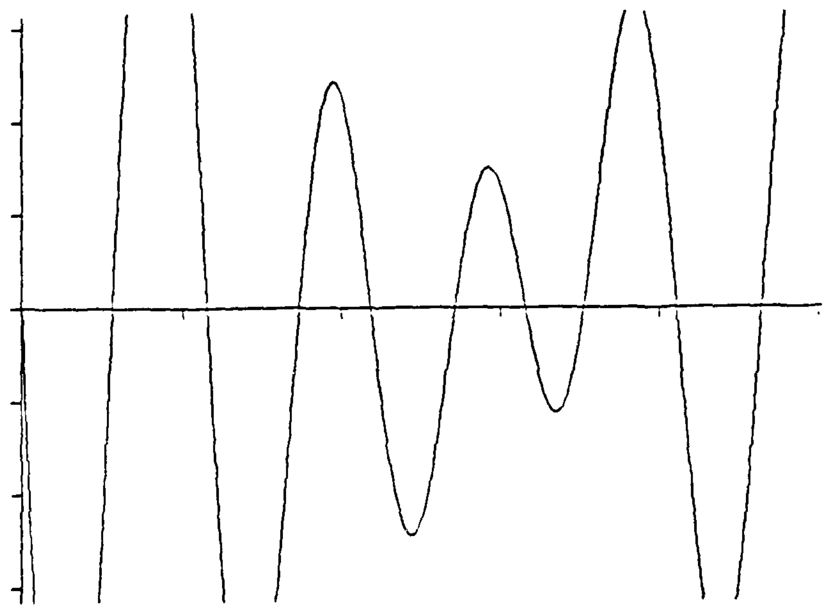


Fig. 84. Example 7 structure y displacement/time K=2

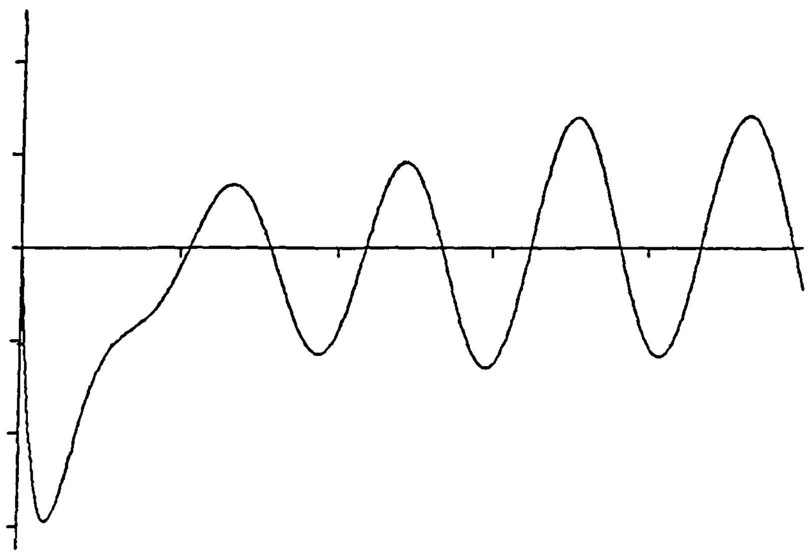


Fig. 85. Example 7 structure y displacement/time K=4

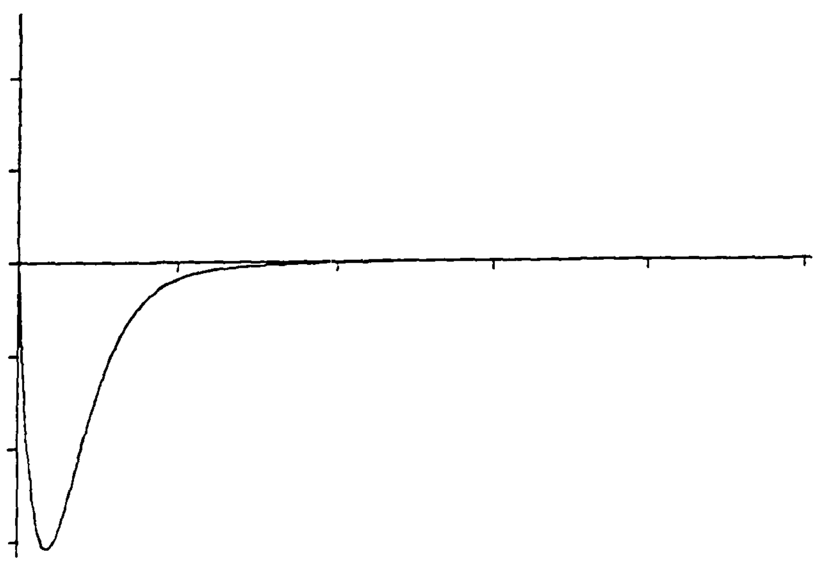


Fig. 86. Example 7 structure y displacement/time K=6

CHAPTER IV

CONCLUSIONS

In this thesis we have been examining a new approach forwarded in [2] for controlling large flexible structures. The importance of this new approach is that by using the independent modal-space control method and a finite number of spatially distributed input points one can essentially eliminate control spillover into modes beyond the reduced order model of the system. By examining the formulation of this control method we see that the control of the system is accomplished through designing the input control forces in the form of equation (56). This spatial modal input-distribution control redistributes the spatially discrete inputs so as to create effective continuously distributed damping and stiffness inputs. The inherent spatial orthogonal non-pass filtering property of this control results in the essential elimination of the excitation effect of the residual modes.

In Chapter II and III we applied the closed-loop control method to an undamped simply supported beam and an undamped truss. By examining the displacement responses of the reduced order model and the structure we can see

that as the number of inputs increase the reduced order model approached the ideal model case where the effect of residual mode excitation is eliminated from the structure. If we compare the reduced order model's gain matrix and closed-loop eigenvalues to those of the ideal model we can see that the reduced order model approaches the ideal model with a finite number of inputs. By looking at the input force responses we can see that the input force is reduced with the increase of inputs until it reaches the ideal input force response which represents a continuously distributed input force. We also applied an open-loop control system to the truss problem and by examining the displacement responses we can see that with enough input points the residual mode excitation can be eliminated just as effectively as the closed-loop control system. It is very important to note that higher numbers of input points were examined for each of the examples shown, but the effect on the response plots is so small that it is impossible to see.

From these examples it is easy to see that by using spatial modal input-distribution control and a finite number of inputs it is possible to essentially eliminate control spillover. Therefore, the method forwarded in [2] can be a very useful approach in controlling the large flexible space structures of the future.

APPENDIX A

BEAM CONTROL SIMULATION PROGRAM

```
//  JOB  ,
//   TIME=(2,0),REGION=2560K
/*JOBPARM  LINES=9000
//  EXEC PLOTV77
//SYSLIB DD DSN=SYS1.VFORTLIB,DISP=SHR
//           DD DSN=SYS1.PLOTLIB,DISP=SHR
//           DD DSN=SYS1.FORTLIB,DISP=SHR
//           DD DSN=SYS1.IMSL.DOUBLE,DISP=SHR
//GO.SOURCE DD *
C***********************************************
C*                                             *
C*           PROGRAM  PNTCNT                   *
C*                                             *
C***********************************************
      IMPLICIT REAL*8(A-H,O-Z)
      EXTERNAL F1,F2
      DOUBLE PRECISION CF
      DIMENSION C11(40,40),C(40,40),CCDUM(40,40),WCS(40),
     *CCINV(40,40),WKAREA(500),A(40,40),BCB2(40,40),
     *WC(40),WSTS(40),WSTINS(40),GM(40,40),PHIM(40,40),
     *G(40,40),ALPHA(40),OMEGA(40),BETA(40),B(40,40),
     *GAMS(40,40),GAMM(40,40),BS(40,40),BCB(40,40),CBW(40),
     *PHIS(40,40),BT(40,40),CINVBT(40,40),WSTINM(40),
     *GMN(40,40),ACLI(40,40),ACLN(40,40),WK(1700),WSTM(40)
      COMPLEX*16 EVALI(40),EVALN(40),EVECI(40,40),
     *ZNI,ZNN,EVECN(40,40)
      COMMON X(40),STIFF,DLEN,DNORM,N,KPM,M,IDUM,DUMJ,PI
      COMMON/COMCIZ/DISP(5002,46),YM(10),YT(10)
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C                                   C
C   INPUT SYSTEM INITIAL CONDITIONS  C
C                                   C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      READ*,  N,NS,K
      READ*,  DT,STIFF,DLEN,RO,NUM
      READ*,  ALPHA1,BETA1
      DO 10 I=1,K
   10 READ*,  X(I)
      N2=N*2
      NS2=NS*2
      LD=40
      KDIM=2*K+1
      DNORM=DSQRT(2.D0/DLEN)
      PI=DATAN(1.D 00)*4.
      IDGT=8
      IJOB=2
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C                               C
C  INITIAL CONDITIONS FOR PLOT  C
C                               C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      YM(1)=30.
```

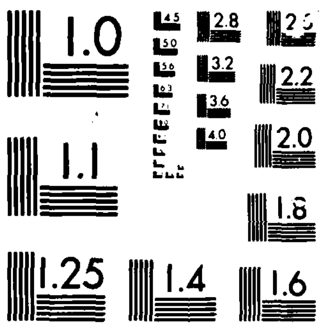
```
      YM(2)=30.
      YM(3)=30.
      YM(4)=30.
      YT(1)=10.
      YT(2)=10.
      YT(3)=10.
      YT(4)=10.
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C                            C
C     INPUT EIGENVALUES      C
C                            C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      DO 15 I=1,NS
      RI=I
   15 OMEGA(I)=RI**2*PI**2*DSQRT(STIFF/RO)/DLEN**2
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C                              C
C   IMSC MODAL GAIN MATRIX     C
C                              C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      DO 16 I=1,N
   16 BETA(I)=BETA1/OMEGA(I)
      DO 17 I=1,N
   17 ALPHA(I)=ALPHA1
      DO 20 I=1,N2
      DO 20 J=1,N2
   20 GM(I,J)=0.
      DO 25 I=1,N
      GM(2*1,2*I-1)=BETA(I)
   25 GM(2*I,2*I)=ALPHA(I)
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C                                     C
C  INPUT MODAL MODEL INITIAL CONDITIONS C
C                                     C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      DO 151 I=1,N2
      READ*, WSTINM(I)
  151 WSTM(I)=WSTINM(I)
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C                                     C
C    STRUCTURE INITIAL CONDITIONS     C
C                                     C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      DO 145 I=1,NS2
      WSTS(I)=0.
  145 WSTINS(I)=0.
      DO 146 I=1,N2
      WSTINS(I)=WSTINM(I)
  146 WSTS(I)=WSTINS(I)
CCCCCCCCCCCCCCCCCCCCCCCCCC
C                        C
C    C MATRIX            C
```

```
C                          C
CCCCCCCCCCCCCCCCCCCCCCCCCCCC
      DO 4 I=1,K
      DO 4 J=1,K
      IF(X(I).LE.X(J)) C(I,J)=CF(X(I),X(J))
    4 IF(X(I).GT.X(J)) C(I,J)=CF(X(J),X(I))
      DO 49 I=1,K
      DO 49 J=1,K
   49 C11(I,J)=C(I,J)
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C                            C
C   B, BT, AND BS MATRICES   C
C                            C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      DO 12 I=1,N
      DO 12 J=1,K
      DI=I
      B(I,J)=DNORM*DSIN(DI*X(J)*PI/DLEN)/OMEGA(I)
   12 BT(J,I)=B(I,J)
      DO 13 I=1,NS
      DO 13 J=1,K
      DI=I
   13 BS(I,J)=DNORM*DSIN(DI*X(J)*PI/DLEN)/OMEGA(I)
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C                                 C
C  C INV. AND BCB MATRICES        C
C                                 C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      CALL LINV2F(C,K,LD,CCINV,IDGT,WKAREA,IER)
      CALL GPROD(CCINV,BT,CINVBT,K,K,N)
      CALL GPROD(BS,CINVBT,BCB,NS,K,N)
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C                                   C
C   TRANSITION & GAMMA MATRICES     C
C                                   C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      DO 500 I=1,NS2
      DO 500 J=1,NS2
      GAMS(I,J)=0.
  500 PHIS(I,J)=0.
      DO 501 I=1,N2
      DO 501 J=1,N2
      GAMM(I,J)=0.
  501 PHIM(I,J)=0.
      DO 502 I=1,NS
      PHIS(2*I-1,2*I-1)=DCOS(OMEGA(I)*DT)
      PHIS(2*I-1,2*I)=DSIN(OMEGA(I)*DT)
      PHIS(2*I,2*I-1)=-PHIS(2*I-1,2*I)
      PHIS(2*I,2*I)=PHIS(2*I-1,2*I-1)
      GAMS(2*I-1,2*I-1)=DSIN(OMEGA(I)*DT)/OMEGA(I)
      GAMS(2*I-1,2*I)=(1.-DCOS(OMEGA(I)*DT))/OMEGA(I)
      GAMS(2*I,2*I-1)=-GAMS(2*I-1,2*I)
```

```
  502 GAMS(2*I,2*I)=GAMS(2*I-1,2*I-1)
      DO 503 I=1,N2
      DO 503 J=1,N2
      PHIM(I,J)=PHIS(I,J)
  503 GAMM(I,J)=GAMS(I,J)
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C                                      C
C    CLOSED LOOP EIGENVALUES & VECTORS C
C                                      C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      DO 201 I=1,N2
      DO 201 J=1,N2
      A(I,J)=0.
      ACLI(I,J)=0.
      ACLN(I,J)=0.
      GMN(I,J)=.0
  201 BCB2(I,J)=0.
      DO 202 I=1,N
      DO 202 J=1,N
  202 BCB2(2*I,2*J)=BCB(I,J)
      CALL GPROD(BCB2,GM,GMN,N2,N2,N2)
      DO 204 I=1,N
      A(2*I-1,2*I)=OMEGA(I)
  204 A(2*I,2*I-1)=-OMEGA(I)
      CALL GMADD(A,GM,ACLI,N2,N2)
      CALL GMADD(A,GMN,ACLN,N2,N2)
      CALL EIGRF(ACLI,N2,LD,IJOB,EVALI,EVECI,LD,WK,IER)
      CALL EIGRF(ACLN,N2,LD,IJOB,EVALN,EVECN,LD,WK,IER)
CCCCCCCCCCCCCCCCCCCCCCCC
C                      C
C    PRINT OUTPUT      C
C                      C
CCCCCCCCCCCCCCCCCCCCCCCC
      WRITE(6,301) N,NS,K
  301 FORMAT(//,2X,'N=',I4,4X,'NS=',I4,4X,'K=',I4)
      WRITE(6,302) DT,STIFF,DLEN,RO,NUM
  302 FORMAT(//,2X,'DT=',E12.4,2X,'STIFF=',E12.4,2X,
     *'DLEN=',E12.4,2X,'RO=',E12.4,2X,'NUM=',I3)
      WRITE(6,303) ALPHA1,BETA1
  303 FORMAT(//,2X,'ALPHA= ',E12.4,4X,'BETA= ',E12.4)
      WRITE(6,304)
  304 FORMAT(//,2X,' X VECTOR ')
      WRITE(6,100) (X(I), I=1,K)
      WRITE(6,305)
  305 FORMAT(//,2X,' WSTINM VECTOR ')
      WRITE(6,100) (WSTINM(I), I=1,N2)
  100 FORMAT(//,2X,7(2X,D16.7))
      WRITE(6,107)
  107 FORMAT(//2X,'C MATRIX')
      DO 21 I=1,K
   21 WRITE(6,100) (C11(I,J),J=1,K)
      WRITE(6,111)
```

```
  111 FORMAT(//,2X,'C INVERSE MATRIX')
      DO 40 I=1,K
   40 WRITE(6,100) (CCINV(I,J),J=1,K)
      WRITE(6,152)
  152 FORMAT(//,2X,' BCB  MATRIX')
      DO 153 I=1,NS
  153 WRITE(6,100) (BCB(I,J),J=1,N)
      WRITE(6,170)
  170 FORMAT(//,2X,'IDEAL GAIN MATRIX')
      DO 171 I=1,N2
  171 WRITE(6,100) (GM(I,J),J=1,N2)
      WRITE(6,172)
  172 FORMAT(//,2X,'NON-IDEAL GAIN MATRIX')
      DO 173 I=1,N2
  173 WRITE(6,100) (GMN(I,J),J=1,N2)
      WRITE(6,161)
  161 FORMAT(//,2X,'COMPLEX CLOSED LOOP EIGENVALUES')
      WRITE(6,160) (EVALN(I),I=1,N2)
  160 FORMAT(//,2X,3(2X,2E16.7))
      WRITE(6,162)
  162 FORMAT(//,2X,'IDEAL COMPLEX CLOSED LOOP
     * EIGENVALUES')
      WRITE(6,160) (EVALI(I),I=1,N2)
      WRITE(6,163)
  163 FORMAT(//,2X,'COMPLEX CLOSED LOOP MODAL MATRIX')
      DO 164 I=1,N2
  164 WRITE(6,160) (EVECN(I,J),J=1,N2)
      WRITE(6,165)
  165 FORMAT(//,2X,'IDEAL COMPLEX CLOSED LOOP MODAL
     * MATRIX')
      DO 166 I=1,N2
  166 WRITE(6,160) (EVECI(I,J),J=1,N2)
      WRITE(6,150)
  150 FORMAT(//,2X, 'PHI MATRIX' )
      DO 154 J=1,NS2
  154 WRITE(6,100)(PHIS(I,J),I=1,NS2)
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C                                           C
C   STRUCTURAL RESPONSE TO MODAL INPUTS     C
C    (WSTS=STRUCTURE MODAL COORDINATES)     C
C                                           C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      TIME=-DT
      KTIME=0
  147 KTIME=KTIME+1
      TIME=TIME+DT
      IF (KTIME.EQ.1) GO TO 510
      DO 138 I=1,N2
      WC(I)=0.
      DO 138 J=1,N2
  138 WC(I)=WC(I)+GM(I,J)*WSTS(J)
      DO 504 I=1,NS2
```

AD-A166 912 CONTROL OF LARGE FLEXIBLE SYSTEMS BY SPATIAL MODAL 2/2
INPUT-DISTRIBUTION CONTROL(U) AIR FORCE INST OF TECH
WRIGHT-PATTERSON AFB OH C V BENDORF 1985
UNCLASSIFIED AFIT/CI/NR-86-47T F/G 22/2 NL



MICROCOPY CHART

```
  504 WCS(I)=0.
      DO 505 I=1,NS
      WCS(2*I)=0.
      DO 505 J=1,N
      CBW(J)=0.
  505 WCS(2*I)=WCS(2*I)+BCB(I,J)*WC(2*J)
      DO 141 I=1,NS2
      WSTS(I)=0.
      DO 141 J=1,NS2
  141 WSTS(I)=WSTS(I)+PHIS(I,J)*WSTINS(J)
      DO 142 I=1,NS2
      DO 142 J=1,NS2
  142 WSTS(I)=WSTS(I)+GAMS(I,J)*WCS(J)
      DO 143 I=1,NS2
  143 WSTINS(I)=WSTS(I)
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C                                    C
C     MODEL RESPONSE TO MODAL INPUTS C
C     (WSTM=MODEL MODAL COORDINATES) C
C                                    C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      DO 136 I=1,N2
      WSTM(I)=0.
      DO 136 J=1,N2
  136 WSTM(I)=WSTM(I)+PHIM(I,J)*WSTINM(J)
      DO 506 I=1,N2
      DO 506 J=1,N2
  506 WSTM(I)=WSTM(I)+GAMM(I,J)*WCS(J)
      DO 137 I=1,N2
  137 WSTINM(I)=WSTM(I)
      DO 180 I=1,K
      DO 180 J=1,N
  180 CBW(I)=CBW(I)+CINVBT(I,J)*WC(2*J)
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C                                 C
C     MODEL & STRUCTURE           C
C     DISPLACEMENT MATRIX         C
C                                 C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
  510 DO 508 J=1,K
      DISP(KTIME,J+1)=0.
      DISP(KTIME,J+1+K)=0.
      DISP(KTIME,J+1+K+K)=0.
      DO 507 I=1,N
      DI=I
  507 DISP(KTIME,J+1)=DISP(KTIME,J+1)+DNORM*DSIN(DI*X(J)
     **PI/DLEN)*WSTM(2*I-1)
      DISP(KTIME,J+1+K+K)=CBW(J)
      DO 509 I=1,NS
      DI=I
  509 DISP(KTIME,J+1+K)=DISP(KTIME,J+1+K)+DNORM*DSIN
     *(DI*X(J)*PI/DLEN)*WSTS(2*I-1)
```

```
      DISP(KTIME, J+1+K)=10.*DISP(KTIME, J+K+1)
      DISP(KTIME, J+1)=10.*DISP(KTIME, J+1)
      DISP(KTIME, J+1+K+K)=10.*DISP(KTIME, J+1+K+K)
  508 CONTINUE
      DISP(KTIME, 1)=TIME
      IF(TIME.LT.2.5D 00) GO TO 147
      WRITE(6,70)
   70 FORMAT(//,2X,'CINVBT MATRIX')
      DO 71 I=1,K
   71 WRITE(6,100) (CINVBT(I,J),J=1,N)
      CALL MPLOT(2.5D 00,0.5D 00,KTIME,K,NUM)
      STOP
      END
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C                             C
C   INFLUENCE FUNCTION (CF)   C
C                             C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      DOUBLE PRECISION FUNCTION CF(Y,ETA)
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON X(40),STIFF,DLEN,DNORM,N,K,M,IDUM,DUMJ,PI
      CF=(Y**3*ETA/DLEN+Y*ETA**3/DLEN+2.D 00*Y*ETA*DLEN
     *-Y**3-3.D 00*Y*ETA**2)/6.D 00/STIFF
      RETURN
      END
CCCCCCCCCCCCCCCCCCCCCCC
C                     C
C  MATRIX PRODUCT     C
C                     C
CCCCCCCCCCCCCCCCCCCCCCC
      SUBROUTINE GPROD(A,B,C,M,N,K)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION A(40,40),B(40,40),C(40,40)
      DO 1 I=1,M
      DO 1 J=1,K
      C(I,J)=0.
      DO 1 L=1,N
    1 C(I,J)=C(I,J)+A(I,L)*B(L,J)
      RETURN
      END
CCCCCCCCCCCCCCCCCCCCCCCC
C                      C
C  MATRIX ADDITION     C
C                      C
CCCCCCCCCCCCCCCCCCCCCCCC
      SUBROUTINE GMADD(A,B,C,N,M)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION A(40,40),B(40,40),C(40,40)
      DO 1 I=1,N
      DO 1 J=1,M
    1 C(I,J)=A(I,J)+B(I,J)
      RETURN
```

```
      END
CCCCCCCCCCCCCCCCCCCCCCCCC
C                       C
C  PLOTTING ROUTINE     C
C                       C
CCCCCCCCCCCCCCCCCCCCCCCCC
      SUBROUTINE MPLOT(XMAG,XT,M,KM,KPT)
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON/COMCIZ/A(5002,46),YM(10),YT(10)
      XSCALE=10./2.5
      XORG=1.
      YORG=5.5
      CALL PLOTS(0,0,0)
      DO 5 I=1,3
      CALL PLOT(1.0,0.,-3)
      CALL PLOT(0.,11.,2)
      CALL AX(5.0D 00,1.0D 00,YM(I),YT(I),XORG,YORG)
      YSCALE=3.5/YM(I)
      K=3
      IF(I.EQ.1) N=KPT+1
      IF(I.EQ.2) N=KM+KPT+1
      IF(I.EQ.3) N=KM+KM+KPT+1
      DO 1 J=1,M
      X=A(J,1)*XSCALE+XORG
      Y=A(J,N)*YSCALE+YORG
      CALL PLOT(X,Y,K)
    1 CONTINUE
      CALL PLOT(10.5,0.,-3)
    5 CONTINUE
      CALL PLOT(0.,0.,999)
      RETURN
      END
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C                            C
C  PLOTTING SUBROUTINE       C
C                            C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      SUBROUTINE AX(XMAG,XT,YMAG,YT,XORG,YORG)
      IMPLICIT REAL*8(A-H,O-Z)
      XM=10.
      YM=3.5
      CALL PLOT(XORG,YORG-YM-.5,3)
      CALL PLOT(XORG,YORG+YM+.5,2)
      Y=YM*YT/YMAG
      YY=YM
      YI=YM+.1
    1 CALL PLOT(XORG,YORG+YY,3)
      CALL PLOT(XORG-.1,YORG+YY,2)
      YY=YY-Y
      IF(YY.GE.-YI) GO TO 1
      X=XM*XT/XMAG
      XX=XM
```

```
      CALL PLOT(XORG,YORG,3)
      CALL PLOT(XORG+XM+.5,YORG,2)
    2 CALL PLOT(XORG+XX,YORG,3)
      CALL PLOT(XORG+XX,YORG-.1,2)
      XX=XX-X
      IF(XX.GT..1) GO TO 2
      RETURN
      END
/*
//GO.DATA DD *

     ( Place Data Here )

/*
//GO.PLOTPARM DD *
 &PLOT  XMAX=120  &END
/*
//
```

APPENDIX B

TRUSS CONTROL SIMULATION PROGRAM

```
//  JOB  ,
//   TIME=(2,0),REGION=2816K
/*JOBPARM  LINES=9000
//  EXEC PLOTV77
//SYSLIB DD DSN=SYS1.VFORTLIB,DISP=SHR
//          DD DSN=SYS1.PLOTLIB,DISP=SHR
//          DD DSN=SYS1.FORTLIB,DISP=SHR
//          DD DSN=SYS1.IMSL.DOUBLE,DISP=SHR
//GO.SOURCE DD *
C***********************************************
C*                                             *
C*          PROGRAM  PNTCNTF                   *
C*            (FINITE MODEL)                   *
C*                                             *
C***********************************************
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION C11(40,40),BSS(40,40),EV(40,40),WCS(40),
     *CCINV(40,40),WKAREA(500),A(40,40),BCB2(40,40),
     *WC(40),WSTS(40),WSTINS(40),GM(40,40),PHIM(40,40),
     *E(40,40),ALPHA(40),OMEGA(40),BETA(40),B(40,40),
     *GAMS(40,40),GAMM(40,40),BS(40,40),BCB(40,40),
     *PHIS(40,40),BT(40,40),CINVBT(40,40),WSTINM(40),
     *GMN(40,40),ACLI(40,40),ACLN(40,40),WK(1700),
     *STIFM(40,40),DMASS(40,40),DK(40,40),DM(40,40),
     *DDK(40,40),DDM(40,40),D(40,40),DMI(40,40),CBW(40),
     *AOL(40,40),OMEGSQ(40),ET(40,40),DMEV(40,40),
     *WSTM(40),DNM(40),DNORM(40)
      COMPLEX*16 EVALI(40),EVALN(40),EVECI(40,40),
     *ZNI,ZNN,EVECN(40,40),CW(40),CEV(40,40)
      COMMON/COMCIZ/DISP(5002,46),YM(10),YT(10)
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C                                     C
C   INPUT SYSTEM INITIAL CONDITIONS   C
C                                     C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
        READ*,  N,NS,K
        READ*,  DT,STIFF,DLEN,RO,NUM
        READ*,  ALPHA1,BETA1
        DO 7 I=1,NS
      7 READ*,  (D(I,J),J=1,K)
        DO 5 I=1,NS
      5 READ*,  (STIFM(I,J),J=1,NS)
        DO 6 I=1,NS
      6 READ*,  (DMASS(I,J),J=1,NS)
        N2=N*2
        NS2=NS*2
        LD=40
        KDIM=2*K+1
        IJOB=2
        IDGT=8
        IJOB=2
        CK=STIFF/(2*DSQRT(2.D 00*DLEN))
```

```
      CM=RO*DLEN**2/(6*DSQRT(2.D 00))
      DO 9 I=1,NS
      DO 9 J=1,NS
      DK(I,J)=CK*STIFM(I,J)
      DM(I,J)=CM*DMASS(I,J)
      DDK(I,J)=DK(I,J)
    9 DDM(I,J)=DM(I,J)
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C                            C
C  INITIAL CONDITIONS FOR PLOT  C
C                            C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      YM(1)=30.
      YM(2)=30.
      YM(3)=30.
      YT(1)=10.
      YT(2)=10.
      YT(3)=10.
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C                                    C
C  INPUT MODAL MODEL INITIAL CONDITIONS  C
C                                    C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      DO 12 I=1,N2
      READ*, WSTINM(I)
   12 WSTM(I)=WSTINM(I)
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C                                     C
C    STRUCTURE INITIAL CONDITIONS      C
C                                     C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      DO 13 I=1,NS2
      WSTS(I)=0.
   13 WSTINS(I)=0.
      DO 14 I=1,N2
      WSTINS(I)=WSTINM(I)
   14 WSTS(I)=WSTINS(I)
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C                            C
C   FIND EIGENVALUES & VECTORS  C
C                            C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      CALL LINV2F(DM,NS,LD,DMI,IDGT,WKAREA,IER)
      CALL GPROD(DMI,DK,AOL,NS,NS,NS)
      CALL EIGRF(AOL,NS,LD,IJOB,CW,CEV,LD,WK,IER)
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C                                 C
C     NORMALIZED MODAL MATRIX     C
C                                 C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      DO 20 I=1,NS
      DNORM(I)=0.
```

```
      OMEGSQ(I)=REAL(CW(I))
      OMEGA(I)=DSQRT(OMEGSQ(I))
      DO 20 J=1,NS
      DMEV(I,J)=0.
   20 EV(I,J)=REAL(CEV(I,J))
      DO 21 JJ=1,NS
      DO 22 I=1,NS
      DO 22 J=1,NS
   22 DMEV(I,JJ)=DMEV(I,JJ)+DDM(I,J)*EV(J,JJ)
      DO 24 I=1,NS
   24 DNORM(JJ)=DNORM(JJ)+EV(I,JJ)*DMEV(I,JJ)
      21 CONTINUE
      DO 26 I=1,NS
      26 DNM(I)=DSQRT(1.D 00/DNORM(I))
      DO 28 I=1,NS
      DO 28 J=1,NS
      28 E(I,J)=DNM(J)*EV(I,J)
CCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C                           C
C   BS, B, AND BT MATRICES  C
C                           C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      DO 30 I=1,NS
      DO 30 J=1,NS
   30 ET(J,I)=E(I,J)
      CALL GPROD(ET,D,BSS,NS,NS,K)
      DO 31 I=1,NS
      DO 31 J=1,K
   31 BS(I,J)=BSS(I,J)/OMEGA(I)
      DO 32 I=1,N
      DO 32 J=1,K
      B(I,J)=BS(I,J)
   32 BT(J,I)=B(I,J)
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C                                C
C  C INV. AND BCB MATRICES       C
C                                C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      CALL AUGMNT(DK,CCINV,NS,K,IDGT,LD)
      CALL GPROD(CCINV,BT,CINVBT,K,K,N)
      CALL GPROD(BS,CINVBT,BCB,NS,K,N)
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C                                C
C  IMSC MODAL GAIN MATRIX        C
C                                C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      DO 42 I=1,N
      BETA(I)=BETA1/OMEGA(I)
   42 ALPHA(I)=ALPHA1
      DO 43 I=1,N2
      DO 43 J=1,N2
   43 GM(I,J)=0.
```

```
      DO 44 I=1,N
      GM(2*1,2*I-1)=BETA(I)
   44 GM(2*I,2*I)=ALPHA(I)
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C                                  C
C   TRANSITION & GAMMA MATRICES     C
C                                  C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      DO 50 I=1,NS2
      DO 50 J=1,NS2
      GAMS(I,J)=0.
   50 PHIS(I,J)=0.
      DO 51 I=1,N2
      DO 51 J=1,N2
      GAMM(I,J)=0.
   51 PHIM(I,J)=0.
      DO 52 I=1,NS
      PHIS(2*I-1,2*I-1)=DCOS(OMEGA(I)*DT)
      PHIS(2*I-1,2*I)=DSIN(OMEGA(I)*DT)
      PHIS(2*I,2*I-1)=-PHIS(2*I-1,2*I)
      PHIS(2*I,2*I)=PHIS(2*I-1,2*I-1)
      GAMS(2*I-1,2*I-1)=DSIN(OMEGA(I)*DT)/OMEGA(I)
      GAMS(2*I-1,2*I)=(1.-DCOS(OMEGA(I)*DT))/OMEGA(I)
      GAMS(2*I,2*I-1)=-GAMS(2*I-1,2*I)
   52 GAMS(2*I,2*I)=GAMS(2*I-1,2*I-1)
      DO 53 I=1,N2
      DO 53 J=1,N2
      PHIM(I,J)=PHIS(I,J)
   53 GAMM(I,J)=GAMS(I,J)
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C                                       C
C    CLOSED LOOP EIGENVALUES & VECTORS   C
C                                       C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      DO 60 I=1,N2
      DO 60 J=1,N2
      A(I,J)=0.
      ACLI(I,J)=0.
      ACLN(I,J)=0.
      GMN(I,J)=.0
   60 BCB2(I,J)=0.
      DO 62 I=1,N
      DO 62 J=1,N
   62 BCB2(2*I,2*J)=BCB(I,J)
      CALL GPROD(BCB2,GM,GMN,N2,N2,N2)
      DO 64 I=1,N
      A(2*I-1,2*I)=OMEGA(I)
   64 A(2*I,2*I-1)=-OMEGA(I)
      CALL GMADD(A,GM,ACLI,N2,N2)
      CALL GMADD(A,GMN,ACLN,N2,N2)
      CALL EIGRF(ACLI,N2,LD,IJOB,EVALI,EVECI,LD,WK,IER)
      CALL EIGRF(ACLN,N2,LD,IJOB,EVALN,EVECN,LD,WK,IER)
```

```
CCCCCCCCCCCCCCCCCCCCCCCC
C                      C
C    PRINT OUTPUT      C
C                      C
CCCCCCCCCCCCCCCCCCCCCCCC
      WRITE(6,301) N,NS,K
  301 FORMAT(//,2X,'N=',I4,4X,'NS=',I4,4X,'K=',I4)
      WRITE(6,302) DT,STIFF,DLEN,RO,NUM
  302 FORMAT(//,2X,'DT=',E12.4,2X,'STIFF=',E12.4,2X,
     *'DLEN=',E12.4,2X,'RO=',E12.4,2X,'NUM=',I3)
      WRITE(6,303) ALPHA1,BETA1
  303 FORMAT(//,2X,'ALPHA= ',E12.4,4X,'BETA= ',E12.4)
      WRITE(6,304)
  304 FORMAT(//2X,'D MATRIX')
      DO 70 I=1,NS
      70 WRITE(6,100) (D(I,J),J=1,K)
      WRITE(6,305)
  305 FORMAT(//,2X,' WSTINM VECTOR ')
      WRITE(6,100) (WSTINM(I), I=1,N2)
  100 FORMAT(//,2X,7(2X,D16.7))
      WRITE(6,306)
  306 FORMAT(//,2X,'STIFFNESS MATRIX')
      DO 85 I=1,NS
   85 WRITE(6,100) (STIFM(I,J),J=1,NS)
      WRITE(6,308)
  308 FORMAT(//,2X,'MASS MATRIX')
      DO 86 I=1,NS
   86 WRITE(6,100) (DMASS(I,J),J=1,NS)
      WRITE(6,311)
  311 FORMAT(//,2X,'C INVERSE MATRIX')
      DO 71 I=1,K
   71 WRITE(6,100) (CCINV(I,J),J=1,K)
      WRITE(6,335)
  335 FORMAT(//,2X,'OPEN LOOP COMPLEX EIGENVALUES')
      WRITE(6,160) (CW(I),I=1,NS)
      WRITE(6,336)
  336 FORMAT(//,2X,'OPEN LOOP COMPLEX EIGENVECTORS')
      DO 72 I=1,NS
   72 WRITE(6,160) (CEV(I,J),J=1,NS)
      WRITE(6,337)
  337 FORMAT(//,2X,' BS MATRIX')
      DO 73 I=1,NS
   73 WRITE(6,100) (BS(I,J),J=1,K)
      WRITE(6,313)
  313 FORMAT(//,2X,' BCB  MATRIX')
      DO 74 I=1,NS
   74 WRITE(6,100) (BCB(I,J),J=1,N)
      WRITE(6,314)
  314 FORMAT(//,2X,'E MATRIX')
      DO 75 I=1,NS
   75 WRITE(6,100) (E(I,J),J=1,NS)
      WRITE(6,315)
```

```
  315 FORMAT(//,2X,'IDEAL GAIN MATRIX')
      DO 76 I=1,N2
   76 WRITE(6,100) (GM(I,J),J=1,N2)
      WRITE(6,317)
  317 FORMAT(//,2X,'NON-IDEAL GAIN MATRIX')
      DO 77 I=1,N2
   77 WRITE(6,100) (GMN(I,J),J=1,N2)
      WRITE(6,316)
  316 FORMAT(//,2X,'COMPLEX CLOSED LOOP EIGENVALUES')
      WRITE(6,160) (EVALN(I),I=1,N2)
  160 FORMAT(//,2X,3(2X,2E16.7))
      WRITE(6,320)
  320 FORMAT(//,2X,'IDEAL COMPLEX CLOSED LOOP EIGENVALUES')
      WRITE(6,160) (EVALI(I),I=1,N2)
      WRITE(6,324)
  324 FORMAT(//,2X,'COMPLEX CLOSED LOOP MODAL MATRIX')
      DO 78 I=1,N2
   78 WRITE(6,160) (EVECN(I,J),J=1,N2)
      WRITE(6,328)
  328 FORMAT(//,2X,'IDEAL COMPLEX CLOSED LOOP MODAL
     *MATRIX')
      DO 82 I=1,N2
   82 WRITE(6,160) (EVECI(I,J),J=1,N2)
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C                                           C
C   STRUCTURAL RESPONSE TO MODAL INPUTS     C
C    (WSTS=STRUCTURE MODAL COORDINATES)     C
C                                           C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      TIME=-DT
      KTIME=0
  147 KTIME=KTIME+1
      TIME=TIME+DT
      IF (KTIME.EQ.1) GO TO 510
      DO 200 I=1,N2
      WC(I)=0.
      DO 200 J=1,N2
  200 WC(I)=WC(I)+GM(I,J)*WSTS(J)
      DO 202 I=1,NS2
  202 WCS(I)=0.
      DO 204 I=1,NS
      WCS(2*I)=0.
      DO 204 J=1,N
      CBW(J)=0.
  204 WCS(2*I)=WCS(2*I)+BCB(I,J)*WC(2*J)
      DO 206 I=1,NS2
      WSTS(I)=0.
      DO 206 J=1,NS2
  206 WSTS(I)=WSTS(I)+PHIS(I,J)*WSTINS(J)
      DO 208 I=1,NS2
      DO 208 J=1,NS2
  208 WSTS(I)=WSTS(I)+GAMS(I,J)*WCS(J)
```

```
      DO 210 I=1,NS2
  210 WSTINS(I)=WSTS(I)
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C                                     C
C    MODEL RESPONSE TO MODAL INPUTS   C
C    (WSTM=MODEL MODAL COORDINATES)   C
C                                     C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      DO 250 I=1,N2
      WSTM(I)=0.
      DO 250 J=1,N2
  250 WSTM(I)=WSTM(I)+PHIM(I,J)*WSTINM(J)
      DO 252 I=1,N2
      DO 252 J=1,N2
  252 WSTM(I)=WSTM(I)+GAMM(I,J)*WCS(J)
      DO 254 I=1,N2
  254 WSTINM(I)=WSTM(I)
      DO 256 I=1,K
      DO 256 J=1,N
  256 CBW(I)=CBW(I)+CINVBT(I,J)*WC(2*J)
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C                                  C
C    MODEL & STRUCTURE             C
C    DISPLACEMENT MATRIX           C
C                                  C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
  510 DO 508 J=1,NS
      DISP(KTIME,J+1)=0.
      DISP(KTIME,J+1+NS)=0.
      DO 507 I=1,N
  507 DISP(KTIME,J+1)=DISP(KTIME,J+1)+E(J,I)*WSTM(2*I-1)
      DO 509 I=1,NS
  509 DISP(KTIME,J+1+NS)=DISP(KTIME,J+1+NS)+E(J,I)*
     *WSTS(2*I-1)
      DISP(KTIME,J+1+NS)=40.*DISP(KTIME,J+NS+1)
      DISP(KTIME,J+1)=40.*DISP(KTIME,J+1)
  508 CONTINUE
      DO 511 J=1,K
  511 DISP(KTIME,J+1+NS+NS)=40*CBW(J)
      DISP(KTIME,1)=TIME
      IF(TIME.LT.10.0D 00) GO TO 147
      CALL MPLOT(10.D 00,2.0D 00,KTIME,NS,NUM)
      STOP
      END
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C                             C
C    MATRIX AUGMENTATION      C
C                             C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      SUBROUTINE AUGMNT(A,B,N,M,IDGT,LD)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION A(40,40),B(40,40),A11(40,40),A12(40,40),
```

```
     *A21(40,40),A22(40,40),AK(40,40),WKAREA(500),
     *AI22(40,40),AIA(40,40)
      NB=N-M
      IF (NB.EQ.0) THEN
         DO 5 I=1,N
         DO 5 J=1,N
    5    B(I,J)=A(I,J)
      ENDIF
      IF (NB.EQ.0) GO TO 10
      DO 1 I=1,M
      DO 1 J=1,M
    1 A11(I,J)=A(I,J)
      DO 2 I=1,M
      DO 2 J=1,NB
    2 A12(I,J)=A(I,M+J)
      DO 3 I=1,NB
      DO 3 J=1,M
    3 A21(I,J)=-A(M+I,J)
      DO 4 I=1,NB
      DO 4 J=1,NB
    4 A22(I,J)=A(M+I,M+J)
      CALL LINV2F(A22,NB,LD,AI22,IDGT,WKAREA,IER)
      CALL GPROD(AI22,A21,AIA,NB,NB,M)
      CALL GPROD(A12,AIA,AK,M,NB,M)
      CALL GMADD(A11,AK,B,M,M)
   10 CONTINUE
      RETURN
      END
CCCCCCCCCCCCCCCCCCCCCCC
C                     C
C  MATRIX PRODUCT     C
C                     C
CCCCCCCCCCCCCCCCCCCCCCC
      SUBROUTINE GPROD(A,B,C,M,N,K)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION A(40,40),B(40,40),C(40,40)
      DO 1 I=1,M
      DO 1 J=1,K
      C(I,J)=0.
      DO 1 L=1,N
    1 C(I,J)=C(I,J)+A(I,L)*B(L,J)
      RETURN
      END
CCCCCCCCCCCCCCCCCCCCCCCC
C                      C
C  MATRIX ADDITION     C
C                      C
CCCCCCCCCCCCCCCCCCCCCCCC
      SUBROUTINE GMADD(A,B,C,N,M)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION A(40,40),B(40,40),C(40,40)
      DO 1 I=1,N
```

```
      DO 1 J=1,M
      1 C(I,J)=A(I,J)+B(I,J)
      RETURN
      END
CCCCCCCCCCCCCCCCCCCCCCC
C                     C
C  PLOTTING ROUTINE   C
C                     C
CCCCCCCCCCCCCCCCCCCCCCC
      SUBROUTINE MPLOT(XMAG,XT,M,KM,KPT)
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON/COMCIZ/A(5002,46),YM(10),YT(10)
      XSCALE=10./10.
      XORG=1.
      YORG=5.5
      CALL PLOTS(0,0,0)
      DO 5 I=1,3
      CALL PLOT(1.0,0.,-3)
      CALL PLOT(0.,11.,2)
      CALL AX(XMAG,XT,YM(I),YT(I),XORG,YORG)
      YSCALE=3.5/YM(I)
      K=3
      IF(I.EQ.1) N=KPT+1
      IF(I.EQ.2) N=KM+KPT+1
      IF(I.EQ.3) N=KM+KM+KPT+1
      DO 1 J=1,M
    4 X=A(J,1)*XSCALE+XORG
      Y=A(J,N)*YSCALE+YORG
    3 CALL PLOT(X,Y,K)
      IF(K.EQ.3) K=2
    1 CONTINUE
    6 CALL PLOT(10.5,0.,-3)
    5 CONTINUE
      CALL PLOT(0.,0.,999)
      RETURN
      END
CCCCCCCCCCCCCCCCCCCCCCCCCCCC
C                          C
C  PLOTTING SUBROUTINE     C
C                          C
CCCCCCCCCCCCCCCCCCCCCCCCCCCC
      SUBROUTINE AX(XMAG,XT,YMAG,YT,XORG,YORG)
      IMPLICIT REAL*8(A-H,O-Z)
      XM=10.
      YM=3.5
      CALL PLOT(XORG,YORG-YM-.5,3)
      CALL PLOT(XORG,YORG+YM+.5,2)
      Y=YM*YT/YMAG
      YY=YM
      YI=YM+.1
    1 CALL PLOT(XORG,YORG+YY,3)
      CALL PLOT(XORG-.1,YORG+YY,2)
```

```
      YY=YY-Y
      IF(YY.GE.-YI) GO TO 1
      X=XM*XT/XMAG
      XX=XM
      CALL PLOT(XORG,YORG,3)
      CALL PLOT(XORG+XM+.5,YORG,2)
    2 CALL PLOT(XORG+XX,YORG,3)
      CALL PLOT(XORG+XX,YORG-.1,2)
      XX=XX-X
      IF(XX.GT..1) GO TO 2
      RETURN
      END
/*
//GO.DATA DD *

    ( Place Data Here )

/*
//GO.PLOTPARM DD *
 &PLOT  XMAX=120  &END
/*
//
```

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